TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 24 No. 4 **2018**

MSC: 42A20

DOI: 10.21538/0134-4889-2018-24-4-104-109

CONVERGENCE OF TRIGONOMETRIC FOURIER SERIES OF FUNCTIONS WITH A CONSTRAINT ON THE FRACTALITY OF THEIR GRAPHS

M. L. Gridnev

For a function f continuous on a closed interval, its modulus of fractality $\nu(f,\varepsilon)$ is defined as the function that maps any $\varepsilon > 0$ to the smallest number of squares of size ε that cover the graph of f. The following condition for the uniform convergence of the Fourier series of f is obtained in terms of the modulus of fractality and the modulus of continuity $\omega(f,\delta)$: if

$$\omega(f,\pi/n)\ln\left(\frac{\nu(f,\pi/n)}{n}\right)\longrightarrow 0 \quad \text{as } n\longrightarrow+\infty,$$

then the Fourier series of f converges uniformly. This condition refines the known Dini–Lipschitz test. In addition, for the growth order of the partial sums $S_n(f,x)$ of a continuous function f, we derive an estimate that is uniform in $x\in[0,2\pi]$:

$$S_n(f,x)=o\biggl(\ln\left(\frac{\nu(f,\pi/n)}{n}\right)\biggr).$$

The optimality of this estimate is shown.

Keywords: trigonometric Fourier series, uniform convergence, fractal dimension.

REFERENCES

1. Gridnev M. L. On classes of functions with a restriction on the fractality of their graphs. In: A. A. Makhnev, S. F. Pravdin (eds.): *Proc. of the 48th Internat. Youth School-Conf. "Modern Problems in Mathematics and its Applications"*, Yekaterinburg, 2017, vol. 1894, pp. 167–173 (in Russian). Published at http://ceur-ws.org/Vol-1894/appr5.pdf .
2. Gridnev M. L. Divergence of Fourier series of continuous functions with restriction on the fractality of their graphs. *Ural Math. J.*, 2017, vol. 3, no. 2, pp. 46–50. doi: 10.15826/umj.2017.2.007 .
3. Bary N.K. *A treatise on trigonometric series*, vol. I; II. Oxford; N Y: Pergamon Press, 1964, 553 p.; 508 p. doi: 10.1002/zamm.19650450531 . Original Russian text published in Bari N.K. *Trigonometricheskie ryady*, Moscow: GIMFL Publ., 1961, 937 p.

Received August 31, 2018
Revised October 28, 2018
Accepted November 05, 2018

Funding Agency: This work was supported by the Russian Science Foundation (project no. 14-11-00702).

Maksim Leonidovich Gridnev, Krasovskii Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620990 Russia, e-mail: coraxcoraxg@gmail.com .

Cite this article as:
M. L. Gridnev. Convergence of trigonometric Fourier series of functions with a constraint on the fractality of their graphs, *Trudy Inst. Mat. Mekh. UrO RAN*, 2018, vol. 24, no. 4, pp.104–109 .