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HIGHEST DIMENSION COMMUTATIVE IDEALS OF A NILTRIANGULAR SUBALGEBRA OF A CHEVALLEY ALGEBRA OVER A FIELD

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Let N be a niltriangular subalgebra of a Chevalley algebra. We study the problem of describing commutative ideals of N of the highest dimension over an arbitrary field. It is proved that N contains a commutative ideal of this dimension, and all such ideals are found. In addition, all maximal commutative ideals of N are described for the types G_2 and F_4 . As a consequence, the highest dimension of commutative subalgebras in all subalgebras of N is found.

Keywords: Chevalley algebra, niltriangular subalgebra, commutative ideals and highest dimension ideals.

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