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ON FINITE SIMPLE LINEAR AND UNITARY GROUPS OF SMALL SIZE OVER FIELDS OF DIFFERENT CHARACTERISTICS WITH COINCIDING PRIME GRAPHS

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Suppose that G is a finite group, $\pi(G)$ is the set of prime divisors of its order, and $\omega(G)$ is the set of orders of its elements. A graph with the following adjacency relation is defined on $\pi(G)$: different vertices r and s from $\pi(G)$ are adjacent if and only if $rs \in \omega(G)$. This graph is called the *Gruenberg–Kegel graph* or the *prime graph* of G and is denoted by $GK(G)$. In A. V. Vasil'ev's Question 16.26 from the "Kourovka Notebook," it is required to describe all pairs of nonisomorphic simple nonabelian groups with identical Gruenberg–Kegel graphs. M. Hagie and M. A. Zvezdina gave such a description in the case where one of the groups coincides with a sporadic group and an alternating group, respectively. The author solved this question for finite simple groups of Lie type over fields of the same characteristic. In the present paper we prove the following theorem.

Theorem. Let $G = A^{\pm}_{n-1}(q)$, where $n \in \{3, 4, 5, 6\}$, and let G_1 be a finite simple group of Lie type over a field of order q_1 nonisomorphic to G, where $q = p^f$, $q_1 = p_1^{f_1}$, and p and p_1 are different primes. If the graphs $GK(G)$ and $GK(G_1)$ coincide, then one of the following statements holds:

(1) $\{G, G_1\} = \{A_1(7), A_1(8)\}$;

(2) $\{G, G_1\} = \{A_3(3), {}^2F_4(2)'\}$;

(3) $\{G, G_1\} = \{{}^2A_3(3), A_1(49)\}$;

(4) $\{G, G_1\} = \{A_2(q), {}^3D_4(q_1)\}$, where $(q-1)_3 \neq 3$, $q+1 \neq 2^k$, and $q_1 > 2$;

(5) $\{G, G_1\} = \{A^{\varepsilon}_4(q), A^{\varepsilon_1}_4(q_1)\}$, where qq_1 is odd;

(6) $\{G, G_1\} = \{A^{\varepsilon}_4(q), {}^3D_4(q_1)\}$, where $(q-\epsilon 1)_5 \neq 5$ and $q_1 > 2$;

(7) $G = A^{\varepsilon}_5(q)$ and $G_1 \in \{B_3(q_1), C_3(q_1), D_4(q_1)\}$.

Keywords: finite simple group of Lie type, prime graph, Gruenberg–Kegel graph, spectrum.

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