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ASYMPTOTIC EXPANSION OF A SOLUTION TO A SINGULAR PERTURBATION OPTIMAL CONTROL PROBLEM WITH A SMALL COERCIVITY COEFFICIENT

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We consider an optimal control problem for solutions of a boundary value problem for a singularly perturbed elliptic operator in a domain Ω with distributed control

$$\mathcal{L}_\varepsilon z_\varepsilon := -\varepsilon^2\Delta z_\varepsilon + a(x)z_\varepsilon = f + u_\varepsilon, \quad x\in\Omega, \quad z_\varepsilon \in H_0^1(\Omega),$$

$$u_\varepsilon \in \mathcal{U} := \{u(\cdot)\in L_2(\Omega)\colon \|u(\cdot)\| \leqslant 1\},$$

$$J := \|z_\varepsilon(\cdot) - z_d(\cdot)\|^2 + \nu^{-1}\|u_\varepsilon(\cdot)\|^2 \to \inf.$$

A priori bounds are obtained for the optimality system, which show that a formal asymptotic solution of the optimality system is an asymptotic expansion of the required solution of this system. A complete asymptotic expansion in the Erdélyi sense in the powers of the small parameter is constructed for the solution of the optimality system for the optimal control problem under consideration. In contrast to the previous papers on this topic, the nonnegative potential $a(\cdot)$ may vanish at a finite number of points. This problem has greater regularity as compared to the problem of studying the asymptotic expansion of the boundary value problem for this operator. The asymptotic expansion consists of an outer power expansion and an inner expansion (in a neighborhood of the boundary of Ω) with exponentially decreasing coefficients.

Keywords: optimal control, asymptotic expansion, singular perturbation problems, small parameter.

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