Vol. 24 No. 3 **2018**

MSC: 30C10, 30C15, 30E10

DOI: 10.21538/0134-4889-2018-24-3-5-15

POLYNOMIALS LEAST DEVIATING FROM ZERO ON A SQUARE OF THE COMPLEX PLANE

E. B. Bayramov

The Chebyshev problem is studied on the square $\Pi = \{z = x + iy \in \mathbb{C}\colon \max\{|x|, |y|\} \leq 1\}$ of the complex plane $\mathbb{C}$. Let $\mathfrak{P}_n$ be the set of algebraic polynomials of a given degree n with the unit leading coefficient. The problem is to find the smallest value $\tau_n(\Pi)$ of the uniform norm $\|p_n\|_{C(\Pi)}$ of polynomials $p_n \in \mathfrak{P}_n$ on the square Π and a polynomial with the smallest norm, which is called the Chebyshev polynomial (for the squire). The Chebyshev constant $\tau(Q) = \lim_{n\to\infty} \sqrt[n]{\tau_n(Q)}$ for the squire is found. Thus, the logarithmic asymptotics of the least deviation $\tau_n(\Pi)$ with respect to the degree of a polynomial is found. The problem is solved exactly for polynomials of degrees from 1 to 7. The class of polynomials in the problem is restricted; more exactly, it is proved that, for $n = 4m + s$, $0 \leq s \leq 3$, it is sufficient to solve the problem on the set of polynomials $z^s q_m(z)$, $q_m \in \mathfrak{P}_m$. Effective two-sided estimates for the value of the least deviation $\tau_n(\Pi)$ with respect to n are obtained.

Keywords: algebraic polynomial, uniform norm, square of the complex plane, Chebyshev polynomial.

REFERENCES

1. Chebyshev P.L. Theory of the mechanisms known as parallelograms. In: Chebyshev P. L. *Collected works. Vol. II. Mathematical analysis.* Moscow; Leningrad: Acad. Sci. USSR, 1947, 520 p., pp. 23–51.
2. Smirnov V.I., Lebedev N.A. *Functions of a complex variable. Constructive theory.* London: Iliffe Books Ltd., 1968, 488 p. Original Russian text published in Smirnov V.I., Lebedev N.A. *Konstruktivnaya teoriya funktsii kompleksnogo peremennogo.* Moscow; Leningrad: Nauka Publ., 1964, 327 p.
3. Milovanovic G.V., Mitrinovic D.S., Rassias Th.M. *Topics in polynomials: Extremal problems, inequalities, zeros.* Singapore: World Scientific Publ. Comp., 1994, 821 p. ISBN: 981-02-0499-X.
4. Thiran J.-P., Detaille C. Chebyshev polynomials on circular arcs in the complex plane. In: *Progress in Approximation Theory*, P. Nevai, A. Pinkus (eds.). Boston, QA: Academic Press, 1991, pp. 771–786. ISBN: 0-12-516750-4.
5. Maergoiz L.S., Rybakova N.N. Chebyshev polynomials with zeros lying on a circular arc. *Dokl. Math.*, 2009, vol. 79, no. 3, pp. 319–321. doi: 10.1134/S1064562409030053.
6. Lukashov A.L., Tyshkevich S.V. Extremal polynomials on arcs of the circle with zeros on these arcs. *J. Contemp. Math. Anal., Armen. Acad. Sci.*, 2009, vol. 44, no. 3, pp. 172–179. doi: 10.3103/S1068362309030030.
7. Lukashov A.L. Inequalities for derivatives of rational functions on several intervals. *Izv. Math.*, 2004. vol. 68, no. 3, pp. 543–565. doi: 10.1070/IM2004v068n03ABEH000488.
8. Arestov V.V., Mendelev A.S. On trigonometric polynomials least deviating from zero. *Dokl. Math.*, 2009, vol. 79, no. 2, pp. 280–283. doi: 10.1134/S1064562409020343.
9. Arestov V.V., Mendelev A.S. Trigonometric polynomials that deviate the least from zero in measure and related problems. *J. Approx. Theory*, 2010, vol. 162, no. 10, pp. 1852–1878. doi: 10.1016/j.jat.2010.07.007.
10. Arestov V.V. Algebraic polynomials least deviating from zero in measure on a segment. *Ukr. Math. J.*, 2010, vol. 62, no. 3, pp. 331–342. doi: 10.1007/s11253-010-0357-z.
11. Babenko A.G. Weak-type inequalities for trigonometric polynomials. *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, 1992, vol. 2, pp. 34–41 (in Russian).

12. Bernstein S.N. *Ekstremal'nye svoistva polinomov i nailuchshee priblizhenie nepreryvnykh funktsii odnoi veshchestvennoi peremennoi* [Extremal properties of polynomials and the best approximation of continuous functions of one real variable]. Moscow; Leningrad: ONTI NKTP SSSR Publ., 1937, 203 p.
13. Bayramov E.B. On Chebyshev polynomials on a square of the complex plane. In: Makhnev A.A. (ed.): *Abstr. 49th Int. Youth School-conf. "Sovremennye problemy matematiki i ee prilozheniya" [Modern Problems in Mathematics and its Applications], Ekaterinburg, Russia, February 4–10, 2018.* Ekaterinburg, 2018, p. 72 (in Russian).
14. Goluzin G.M. *Geometric theory of functions of a complex variable.* Translations of Mathematical Monographs, vol. 26. Providence, R. I.: American Mathematical Society, 1969, 676 p. ISBN: 978-0-8218-1576-2 . Original Russian text published in Goluzin G.M. *Geometricheskaya teoriya funktsii kompleksnogo peremennogo: uchebnoe posobie.* Moscow; Leningrad: Nauka GITTL Publ., 1952, 628 p.
15. Fekete M. Über die Verteilung der Wuzzeln bei gewissen algebraiscnen Gleichugen mit ganzzahligen Koeffizienten. *Math. Z.*, 1923, vol. 17, no. 1, pp. 228–249. doi: 10.1007/BF01504345 .
16. Tobin A.D., Lloyd N.T. *Schwarz-Christoffel mapping: textbook.* Cambridge: Cambridge Univ. Press, 2002, 132 p. ISBN: 9780511029110 .
17. Ivanov V.I., Popov V.Yu. *Konformnye otobrazheniya i ikh prilozheniya* [Conformal mappings and their applications]. Moscow: Editorial URSS Publ., 2002, 324 p. ISBN: 5-354-00178-1 .
18. Volkovyskii L.I., Lunts G.L., Aramanovich I.G. *A Collection of problems on complex analysis.* N Y: Dover Publ., 1991, 426 p. ISBN: 978-0486669137 . Original Russian text published in Volkovyskii L.I., Lunts G.L., Aramanovich I.G. *Sbornik zadach po teorii funktsii kompleksnogo peremennogo.* Moscow, Nauka Publ., 1975, 320 p.
19. Fichtenholz G.M. *Kurs differentsial'nogo i integral'nogo ischisleniya* [A Course in Differential and Integral Calculus] Moscow, Fizmatlit Publ., 2001. Vol. 2. 864 p. (in Russian).

The paper was received by the Editorial Office on July 1, 2018.

Funding Agency: This work was supported by the Russian Foundation for Basic Research (project no. 18-01-00336) and by the Russian Academic Excellence Project (agreement no. 02.A03.21.0006 of August 27, 2013, between the Ministry of Education and Science of the Russian Federation and Ural Federal University).

Emir Batyrovich Bayramov, Ural Federal University, Yekaterinburg, 620002 Russia,
e-mail: mrequ@yandex.ru .

Cite this article as:
E. B. Bayramov. Polynomials least deviating from zero on a square of the complex plan, *Trudy Inst. Mat. Mekh. UrO RAN*, 2018, vol. 24, no. 3, pp. 5–15 .