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PLANCHEREL–PÓLYA INEQUALITY FOR ENTIRE FUNCTIONS OF EXPONENTIAL TYPE IN $L^2(\mathbb{R}^n)$.

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Let $\mathfrak{M}^p_{\sigma,n}$, $p>0$, be a set of entire functions f of n complex variables with exponential type $\sigma=(\sigma_1,\dots,\sigma_n)$, $\sigma_k>0$, such that their restrictions to $\mathbb{R}^n$ belong to $L^p(\mathbb{R}^n)$. In 1937 Plancherel and Pólya showed that $\sum_{k\in\mathbb{Z}^n}|f(k)|^p\leqslant c_p(\sigma,n)\|f\|^p_{L^p(\mathbb{R}^n)}$ for $f\in\mathfrak{M}^p_{\sigma,n}$, where $c_p(\sigma,n)$ is a finite constant. We study the Plancherel–Pólya inequality for $p=2$. If $0<\sigma_k\leqslant\pi$, then, by the Whittaker–Kotelnikov–Shannon theorem and its generalization to the multidimensional case established by Plancherel and Pólya, we have $c_2(\sigma,n)=1$ and any function $f\in\mathfrak{M}^2_{\sigma,n}$ is extremal. In the general case, we prove that $c_2(\sigma,n)=\prod_{k=1}^n\lceil\sigma_k/\pi\rceil$ and describe the class of extremal functions. We also write the dual problem $\big|\sum_{k\in\mathbb{Z}^n}(g*g)(k)\big|\leqslant d_2(\sigma,n)\|g\|_2^2$, $g\in L^2(\Omega)$, prove that $c_2(\sigma,n)=d_2(\sigma,n)$, and describe the class of extremal functions.

Keywords: Plancherel–Pólya inequality, Paley–Wiener space, entire function of exponential type, Fourier transform.

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