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**SOLVABILITY ISSUES FOR A CLASS OF CONVOLUTION
TYPE NONLINEAR INTEGRAL EQUATIONS IN \mathbb{R}^n** **Kh. A. Khachatryan, H. S. Petrosyan, M. H. Avetisyan**

We study a class of nonlinear multidimensional integral equations of convolution type. This class of equations is directly applied in the p -adic theory of open–closed strings. We prove the existence of an n -parametric family of nontrivial continuous bounded solutions and establish certain properties of the constructed solutions: monotonicity in each argument, limit relations, and integral asymptotics. The solutions are used to study a nonlinear problem for the multidimensional heat equation. At the end of the paper we give examples of such equations, which are of independent theoretical and practical interest.

Keywords: nontrivial solution, monotonicity, p -adic theory, limit, successive approximations.

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