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## POLYNOMIAL TIME APPROXIMATION SCHEME FOR THE CAPACITATED VEHICLE ROUTING PROBLEM WITH TIME WINDOWS

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The capacitated vehicle routing problem with time windows (CVRPTW) is a well-known NP-hard combinatorial optimization problem. We present a further development of the approach first proposed by M. Haimovich and A. H. G. Rinnooy Kan and propose an algorithm that, for an arbitrary  $\varepsilon > 0$ , finds a  $(1 + \varepsilon)$ -approximate solution for the Euclidean CVRPTW in  $\text{TIME}(\text{TSP}, \rho, n) + O(n^2) + O(e^{O(q(\frac{q}{\varepsilon})^3(p\rho)^2 \log(p\rho))})$ , where  $q$  is an upper bound for the capacities of the vehicles,  $p$  is the number of time windows, and  $\text{TIME}(\text{TSP}, \rho, n)$  is the complexity of finding a  $\rho$ -approximation solution of an auxiliary instance of the Euclidean TSP. Thus, the algorithm is a polynomial time approximation scheme for CVRPTW with  $p^3 q^4 = O(\log n)$  and an efficient polynomial time approximation scheme (EPTAS) for arbitrary fixed values of  $p$  and  $q$ .

Keywords: capacitated vehicle routing problem, time windows, efficient polynomial time approximation scheme.

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