TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 24 No. 3 **2018**

MSC: 41A15

DOI: 10.21538/0134-4889-2018-24-3-200-225

EXTREMAL FUNCTION INTERPOLATION AND SPLINES

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The paper is a survey of the results obtained in the problems of extremal function interpolation over the past 50 years. Various statements of problems in this direction are analyzed both for the case of one variable and for the case of several variables. A special focus is put on the role of interpolation splines of different types (polynomial, interpolating in the mean, $\mathcal{L}$-splines, m-harmonic, etc.) in solving the problems of extremal function interpolation. Important applications of the results and methods of extremal interpolation to other problems in approximation theory and the theory of splines are specified.

Keywords: interpolation, splines, approximation, differential operators, difference operators.

REFERENCES

1. Gel'fond M.G. *Calculus of finite differences*. Delhi, Hindustan Publishing Corp., 1971, Ser. International Monographs on Advanced Mathematics and Physics, 451 p. Original Russian text published in *Ischislenie konechnykh raznostei*, Moscow: Nauka, 1967.
2. Krein M.G. Integral equations on the half-line with a kernel depending on the difference of the arguments. *Uspehi Mat. Nauk*, 1958, vol. 13, no. 5(83), pp. 3–120 (in Russian).
3. Markushevich A.I. *The theory of analytic functions: a brief course*. Moscow: Mir, 1983, 423 p. Original Russian text published in *Kratkii kurs teorii analiticheskikh funktsii*, Moscow, Nauka, 1966.
4. Novikov S.I., Shevaldin V.T. A problem of extremal interpolation for multivariate functions. *Proc. Steklov Inst. Math.*, 2001, Suppl. 1, S150–S166.
5. Novikov S.I. Periodic interpolation with minimal norm of mth derivative. *Sib. Zh. Vychisl. Math.*, 2006, vol. 9, no. 2, pp. 165–172 (in Russian).
6. Novikov S.I. Interpolation with minimal norm of the Laplace operator in a ball. *Zbirnik prats Inst. Math. NAN Ukraine*, 2008, vol. 5, no. 1. pp. 248–262 (in Russian).
7. Novikov S.I. Interpolation problems with minimal norm of the Laplace operator on a class of interpolated data. *Proc. Intern. Summer Math. S. B. Stechkin Workshop-Conf. on Function Theory*. Dushanbe: Ofset Publ., 2016, pp. 182–185 (in Russian).
8. Patsko N.L. Approximation on an interval by splines. *Math. Notes*, 1974, vol. 16, no. 3, pp. 881–887.
9. Patsko N.L. Approximation by splines on an interval in the space L_p. *Math. Notes*, 1995, vol. 58, no. 1-2, pp. 867–876.
10. Rjabenki V.S., Filippov F.F. *Über die Stabilität von Differenzengleichungen. Mathematik für Naturwissenschaft und Technik.* Bd. 3. Berlin, VEB Deutscher Verlag der Wissenschaften, 1960, 136 p. (in German).
11. Sobolev S.L. *Lektsii po teorii kubaturnykh formul*. [Lectures on the theory of cubature formulas]. Part 2. Novosibirsk: Novosibirsk Gos. Univ., 1965, 192 p. (in Russian).
12. Stechkin S.B., Subbotin Yu.N. *Splainy v vychislitel'noi matematike*. [Splines in numerical mathematics]. Moscow, Nauka, 1976, 248 p. (in Russian).
13. Subbotin Yu.N. On the connection between finite differences and corresponding derivatives. *Tr. Math. Inst. Steklov*, 1965, vol. 78, pp. 24–42 (in Russian).
14. Subbotin Yu.N. Functional interpolation in the mean with smallest nth derivative. *Tr. Math. Inst. Steklov*, 1967, vol. 88, pp. 30–60 (in Russian).

15. Subbotin Yu.N. Approximation by spline functions, and estimates of widths. *Tr. Math. Inst. Steklov*, 1971, vol. 109, pp. 35–60 (in Russian).
16. Subbotin Yu.N. Spline approximation and smooth bases in $C(0, 2\pi)$. *Math. Notes*, 1972, vol. 12, no. 1, pp. 459–463.
17. Subbotin Yu.N. Extremal functional interpolation and splines. *Soviet Math. Dokl.*, 1974, vol. 15, pp. 57–60.
18. Subbotin Yu.N. Extremal problems of functional interpolation, and mean interpolation splines. *Tr. Math. Inst. Steklov*, 1975, vol. 138, pp. 118–173 (in Russian).
19. Subbotin Yu.N. Extremal and approximate properties of splines. *The theory of approximation of functions* (*Proc. Internat. Conf., Kaluga, 1975*), Moscow, Nauka, 1977, pp. 341–345 (in Russian).
20. Subbotin Yu.N. Extremal functional interpolation in the mean with the smallest value of the nth derivative for large averaging intervals. *Math. Notes*, 1996, vol. 59, no. 1-2, pp. 83–96.
21. Subbotin Yu.N. Extremal L_p-interpolation in the mean for intersecting intervals of averaging. *Izv. Math.*, 1997, vol. 61, no. 1, pp. 183–205.
22. Timofeev V.G. Inequalities of Kolmogorov type with the Laplace operator. Theory of functions and approximations, Saratov, Saratov. Gos. Univ., 1983, vol. 106, pp. 84–92 (in Russian).
23. Tihomirov V.M., Bojanov B.D. Some convex problems of approximation theory. *Serdica*, 1979, vol. 5, no. 1, pp. 83–96 (in Russian).
24. Chui C.K. An introduction to wavelets. Boston: Acad. Press, 1992, Ser. Wavelet Analysis and its Applications, 264 p.
25. Sharma A., Tzimbalario J. Certain linear differential operators and generalized differences. *Math. Notes*, 1977, vol. 21, no. 2, pp. 91–97. doi:10.1007/BF02320546 .
26. Shevaldin V.T. Extremal interpolation with least norm of linear differential operator. *Math. Notes*, 1980, vol. 27, no. 5, pp. 344–354.
27. Shevaldin V.T. A problem of extremal interpolation. *Math. Notes*, 1981, vol. 29, no. 4, pp. 310–320.
28. Shevaldin V.T. Some problems of extremal interpolation in the mean. *Soviet Math. Dokl.*, 1982, vol. 26, no. 3, pp. 710–712.
29. Shevaldin V.T. Some problems of extremal interpolation in the mean for linear differential operators. *Tr. Math. Inst. Steklov*, 1983, vol. 164, pp. 203–240 (in Russian).
30. Shevaldin V.T. $\mathcal{L}$-splines and widths. *Math. Notes*, 1983, vol. 33, no. 5, pp. 735–744.
31. Shevaldin V.T. Lower bounds for the widths of some classes of periodic functions. *Proc. Steklov Inst. Math.*, 1994, no. 1 (198), pp. 233–255.
32. Shevaldin V.T. Extremal interpolation in the mean with overlapping intervals of averaging, and L-splines. *Izv. Math.*, 1998, vol. 62, no. 4, pp. 833–856.
33. Atteia M. Functions «spline», definies sur un ensemble convexe. *Numer. Math.*, 1968, vol. 12, pp. 192–210.
34. Behforooz H. Approximation by integro cubic splines. *Appl. Math. Comput.*, 2006, vol. 175, pp. 8–15. doi:10.1016/j.amc.2005.07.066 .
35. de Boor C. How small can one make the derivatives of an interpolating function? *J. Approx. Theory.*, 1990, vol. 13, no. 2, pp. 105–116. doi: 10.1016/0021-9045(75)90043-X .
36. de Boor C., Höllig K., Riemenschneider S. Box splines. N Y etc.: Springer, 1993. 200 p.
37. Burenkov V.I. Sobolev spaces on domains. Stuttgart: B. G. Teubner Verlag GmbH, 1998, Ser. Teubner Texts in Math., vol. 137, 312 p.
38. de Concini C., Procesi C. Topics in hyperplane arrangements, polytopes and box-splines. N Y etc.: Springer, 2010. 384 p.
39. Favard J. Sur l'interpolation. *J. Math. Pures Appl.*, 1940, vol. 19, no 9, p. 281–306.
40. Fisher S., Jerome J. Minimum norm extremals in function spaces. *Lecture Notes in Math.*, 1975, vol. 479, pp. 1–209.
41. Holmes R. Geometric functional analysis and its applications. N.Y. ect.: Springer Verlag, 1975, 246 p.
42. Kunkle T. Favard's interpolation problem in one or more variables. *Constr. Approx.*, 2002, vol. 18, no. 4, pp. 467–478. doi:10.1007/s00365-001-0015-7 .
43. Madych W.R., Nelson S.A. Polyharmonic cardinal splines. *J. Approx. Theory*, 1990, vol. 60, no. 2, pp. 141–156. doi:10.1016/0021-9045(90)90079-6 .
44. Madych W.R., Nelson S.A. Polyharmonic cardinal splines: a minimization property. *J. Approx. Theory*, 1990, vol. 63, no. 3, pp. 303–320. doi:10.1016/0021-9045(90)90123-8 .

45. Micchelli C.A. Cardinal $\mathcal{L}$-splines. Studies in spline functions and approximation theory. N Y: Acad. Press, 1976, pp. 203–250.
46. Novikov S.I. Generalization of the Rolle theorem. *East J. Approx.*, 1995, vol. 1, no. 4, pp. 571–575.
47. Schoenberg I.J. Cardinal interpolation and spline functions. *J. Approx. Theory*, 1969, vol. 2, no. 2, pp. 167–206. doi:10.1016/0021-9045(69)900040-9 .
48. Schoenberg I.J. On Micchelli's theory of cardinal L-splines. *Studies in spline functions and approximation theory*. N Y: Acad. Press, 1976, pp. 251–276.
49. Sharma A., Tzimbalario J. A generalization of a result of Subbotin. *Approximation theory - ll* (*Proc. Internat. Sympos., Univ. Texas, 1976*), N Y: Acad. Press, 1976, pp. 557–562.
50. Subbotin Yu.N. Some extremal problems of interpolation and interpolation in the mean. *East J. Approx.*, 1996, vol. 2, no. 2, pp. 155–167.
51. Zhanlav T., Mijiddorj R. Integro quintic splines and their approximation properties. *Appl. Math. Comput.*, 2014, vol.231, pp. 536–543. doi: 10.1016/j.amc.2014.01.043 .
52. Zhanlav T., Mijiddorj R., Behforooz H.. Construction of local integro quintic splines. *Commun. Numer. Anal.*, 2016, no. 2, pp. 167–179. doi:10.5899/2016/cna-00267 .
53. Zhanlav T., Mijiddorj R. Convexity and monotonicity properties of the local integro cubic spline. *Appl. Math. Comput.*, 2017, vol. 293, pp. 131–137. doi:10.1016/j.amc.2016.08.017 .

The paper was received by the Editorial Office on May 20, 2018.

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Cite this article as:
Yu. N. Subbotin, S. I. Novikov, V. T. Shevaldin. Extremal function interpolation and splines, *Trudy Inst. Mat. Mekh. UrO RAN*, 2018, vol. 24, no. 3, pp. 200–225 .