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ONE APPROACH TO THE SOLUTION OF SOME PROBLEMS IN PLASMA DYNAMICS

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A system of equations for the motion of an ionized ideal gas is considered. An algorithm for the reduction of this system of nonlinear partial differential equations (PDEs) to systems of ordinary differential equations (ODEs) is presented. It is shown that the independent variable ψ in the systems of ODEs is determined from the relation $\psi = t + xf_1(\psi) + yf_2(\psi) + zf_3(\psi)$ after choosing (setting or finding) the functions $f_i(\psi)$, $i = 1, 2, 3$. These functions are either found from the conditions of the problem posed for the original system of PDEs or are given arbitrarily to obtain a specific system of ODEs. For the problem on the motion of an ionized gas near a body, we write a system of ODEs and discuss the issue of instability, which is observed in a number of cases. We also consider a problem of the motion of flows (particles) in a given direction, which is of significant interest in some areas of physics. We find the functions $f_i(\psi)$, $i = 1, 2, 3$, that provide the motion of a flow of the ionized gas in a given direction and reduce the system of PDEs to a system of ODEs.

Keywords: nonlinear partial differential equations, exact solutions, systems of ordinary differential equations, boundary value problem.

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