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COCONVEX INTERPOLATION BY SPLINES WITH THREE-POINT RATIONAL INTERPOLANTS

A.-R. K. Ramazanov, V. G. Magomedova

For discrete functions $f(x)$ defined on arbitrary grid nodes $\Delta : a = x_0 < x_1 < \cdots < x_N = b$ $(N \geqslant 3)$, we study the issues of preserving the (upward or downward) convexity and coconvexity with a change of convexity direction by rational spline-functions $R_{N,1}(x) = R_{N,1}(x, f, \Delta, g(t)) = (R_i(x)(x-x_{i-1})+R_{i-1}(x)(x_i-x))/(x_i-x_{i-1})$, where $x \in [x_{i-1}, x_i]$ $(i = 1, 2, \ldots, N)$, $R_i(x) = \alpha_i + \beta_i(x - x_i) + \gamma_i/(x - g_i(t))$ $(i = 1, 2, \ldots, N-1)$, and $R_i(x_j) = f(x_j)$ $(j = i-1, i, i+1)$. The location of the pole $g_i(t)$ with respect to the nodes x_{i-1} and x_i is defined by the parameter t. We assume that $R_0(x) \equiv R_1(x)$ and $R_N(x) \equiv R_{N-1}(x)$. For these spines we derive the conditions $1/2 < |q_i| < 2$ of convexity preservation, where $q_i = f(x_{i-2}, x_{i-1}, x_i)/f(x_{i-1}, x_i, x_{i+1})$ for $i = 2, 3, \ldots, N-1$.

Keywords: interpolation spline, rational spline, coconvex interpolation, shape-preserving interpolation.

REFERENCES

1. Schweikert D.G. An interpolation curve using a spline in tension. *J. Math. Phys.*, 1966, vol. 45, iss. 1–4, pp. 312–317. doi: 10.1002/sapm1966451312 .
2. Miroshnichenko V.L. Convex and monotone spline interpolation. *Constuctive Theory of Function: Proc. Int. Conf.*, (Varna, 1984), Sofia: Publ. House of Bulgarian Acad. Sci., 1984, pp. 610–620.
3. Miroschnichenko V.L. Sufficient conditions for monotonicity and convexity of cubic splines of class C^2. *Sib. Adv. Math.*, 1992, vol. 2, no. 4, pp. 147–163.
4. Kvasov B.I. *Metody izogeometricheskoi approksimatsii splainami* [Methods of Shape-Preserving Spline Approximaton]. Moscow: Fizmatlit Publ., 2006, 360 p. ISBN: 5-9221-0733-X .
5. Volkov Yu.S., Bogdanov V.V., Miroschnichenko V.L., Shevaldin V.T. Shape-preserving interpolation by cubic splines. *Math. Notes*, 2010, vol. 88, no. 6, pp. 798–805. doi: 10.1134/S0001434610110209 .
6. Schaback R. Spezielle rationale Splinefunktionen. *J. Approx.Theory*, 1973, vol. 7, no. 2. pp. 281–292. doi: 10.1016/0021-9045(73)90072-5 .
7. Spath H. *Spline algorithms for curves and surfaces*. Winnipeg: Utilitas Mathematica Publ. Inc., 1974, 198 p. ISBN: 0919628990 .
8. Hussain M.Z., Sarfraz M., Shaikh T.S. Shape preserving rational cubic spline for positive and convex data. *Egyptian Informatics J.*, 2011, vol. 12, pp. 231–236. doi: 10.1016/j.eij.2011.10.002 .
9. Edeo A., Gofeb G., Tefera T. Shape preserving C^2 rational cubic spline interpolation. *American Sci. Research J. Engineering, Technology and Sciences*, 2015, vol. 12, no. 1. pp. 110–122.
10. Ramazanov A.-R.K., Magomedova V.G. Splines for rational interpolants. *Dagestan. Elektron. Mat. Izv.*, 2015, no. 4, pp. 21–30 (in Russian).
11. Ramazanov A.-R.K., Magomedova V.G. Splines for three-point rational interpolants. *Tr. Matem. centra im. N.I. Lobachevskogo*. Kazan, 2017, vol. 54, pp. 304–306 (in Russian).
12. Ramazanov A.-R.K., Magomedova V.G. Splines for three-point rational interpolants with autonomous poles. *Dagestan. Elektron. Mat. Izv.*, 2017, no. 7, pp. 16–28 (in Russian).
13. Ramazanov A.-R.K., Magomedova V.G. Unconditionally convergent interpolational rational splines. *Math. Notes*, 2018, vol. 103, no. 4, pp. 588–599 (in Russian). doi: 10.4213/mzm11201 .

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Abdul-Rashid Kehrimanovich Ramazanov, Dr. Phys.-Math., Prof., Dagestan State University, the Republic of Dagestan, Makhachkala, 367002 Russia; Dagestan Scientific Center RAN, the Republic of Dagestan, Makhachkala, 367025 Russia, e-mail: ar-ramazanov@rambler.ru .

Vazipat Gusenovna Magomedova, Cand. Sci. (Phys.-Math.), Dagestan State University, the Republic of Dagestan, Makhachkala, 367002 Russia, e-mail: vazipat@rambler.ru .

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