SHILLA DISTANCE-REGULAR GRAPHS WITH $b_2 = sc_2$

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A Shilla graph is a distance-regular graph Γ of diameter 3 whose second eigenvalue is $a = a_3$. A Shilla graph has intersection array $\{ab, (a+1)(b-1), b_2; 1, c_2, a(b-1)\}$. J. Koolen and J. Park showed that, for a given number b, there exist only finitely many Shilla graphs. They also found all possible admissible intersection arrays of Shilla graphs for $b \in \{2, 3\}$. Earlier the author together with A. A. Makhnev studied Shilla graphs with $b_2 = c_2$. In the present paper, Shilla graphs with $b_2 = sc_2$, where s is an integer greater than 1, are studied. For Shilla graphs satisfying this condition and such that their second nonprincipal eigenvalue is -1, five infinite series of admissible intersection arrays are found. It is shown that, in the case of Shilla graphs without triangles in which $b_2 = sc_2$ and $b < 170$, only six admissible intersection arrays are possible. For a Q-polynomial Shilla graph with $b_2 = sc_2$, admissible intersection arrays are found in the cases $b = 4$ and $b = 5$, and this result is used to obtain a list of admissible intersection arrays of Shilla graphs for $b \in \{4, 5\}$ in the general case.

Keywords: distance-regular graph, graph automorphism.

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