ON THE PERMUTABILITY OF A SYLOW SUBGROUP
WITH SCHMIDT SUBGROUPS FROM A SUPPLEMENT

V. S. Monakhov, E. V. Zubei

A Schmidt group is a finite nonnilpotent group each of whose proper subgroups is nilpotent. A supplement of a subgroup A in a group G is a subgroup B of G such that $G = AB$. Finite groups in which a Sylow subgroup is permutable with some Schmidt subgroups were studied by Ya. G. Berkovich and E. M. Pal'chik (Sib. Mat. Zh. **8** (4), 741–753 (1967)) and by V. N. Knyagina and V. S. Monakhov (Proc. Steklov Inst. Math. **272** (Suppl. 1), S55–S64 (2011)). In this situation, the group may be nonsolvable. For example, in the group PSL(2,7) a Sylow 2-subgroup is permutable with all Shmidt subgroups of odd order. In the group SL(2,8) a Sylow 3-subgroup is permutable with all 2-closed Shmidt subgroups of even order. In the group SL(2,4) a Sylow 5-subgroup is permutable with every 2-closed Shmidt subgroup of even order. Since the groups Sz(2^{2k+1}) for $k \geq 1$, PSU(5,4), PSU(4,2), and PSp(4,2^n) do not contain Shmidt subgroups of odd order, in these groups any Sylow subgroup is permutable with any Shmidt subgroup of odd order. We establish the r-solvability a finite group G such that r is odd and is not a Fermat prime and a Sylow r-subgroup R is permutable with 2-nilpotent (or 2-closed) Schmidt subgroups of even order from some supplement of R in G. We give examples showing that the constraints on r are not superfluous.

Keywords: finite group, Schmidt group, r-solvable group, Sylow r-subgroup.

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Viktor Stepanovich Monakhov, Dr. Phys.-Math. Sci., Prof., Francisk Skorina Gomel State University, Gomel, 246019, Republic of Belarus, e-mail: victor.monakhov@gmail.com .

Ekaterina Vladimirovna Zubei, doctoral student, Francisk Skorina Gomel State University, Gomel, 246019, Republic of Belarus, e-mail: ekaterina.zubey@yandex.ru .