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INVERSE PROBLEMS IN THE THEORY OF DISTANCE-REGULAR GRAPHS

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For a distance-regular graph Γ of diameter 3, the graph Γ_i can be strongly regular for $i = 2$ or 3. Finding the parameters of Γ_i given the intersection array of Γ is a direct problem, and finding the intersection array of Γ given the parameters of Γ_i is the inverse problem. The direct and inverse problems were solved earlier by A. A. Makhnev and M. S. Nirova for $i = 3$. In the present paper, we solve the inverse problem for $i = 2$: given the parameters of a strongly regular graph Γ_2, we find the intersection array of a distance-regular graph Γ of diameter 3. It is proved that Γ_2 is not a graph in the half case. We also refine Nirova's results on distance-regular graphs Γ of diameter 3 for which Γ_2 and Γ_3 are strongly regular. New infinite series of admissible intersection arrays are found: $\{r^2 + 3r + 1, r(r + 1), r + 2; 1, r + 1, r(r + 2)\}$ for odd r divisible by 3 and $\{2r^2 + 5r + 2, r(2r + 2), 2r + 3; 1, 2r + 2, r(2r + 3)\}$ for r indivisible by 3 and not congruent to ± 1 modulo 5.

Keywords: strongly regular graph, distance-regular graph, intersection array.

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