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ON A SINGULARLY PERTURBED TIME-OPTIMAL CONTROL PROBLEM WITH TWO SMALL PARAMETERS

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In this paper we investigate a time-optimal control problem for a singularly perturbed linear autonomous system with two independent small parameters and smooth geometric constraints on the control in the form of a ball. The main difference of this case from the systems with fast and slow variables studied earlier is that here the matrix at the fast variables is a multidimensional analog of the second-order Jordan cell with zero eigenvalue and, thus, does not satisfy the standard condition of asymptotic stability. Continuing the research, we consider initial conditions depending on the second small parameter; in the degenerate case, this resulted in an asymptotic expansion of the solution of a fundamentally different type. The solvability of the problem is proved. We also derive and justify a complete power asymptotic expansion in the sense of Erdelyi of the optimal time and optimal control with respect to a small parameter at the derivatives in the equations of the systems.

Keywords: optimal control, time-optimal control problem, asymptotic expansion, singularly perturbed problem, small parameter.

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