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ON THE SOLUTION OF A DIFFERENTIAL GAME OF MANAGING THE INVESTMENTS IN AN ADVERTISING CAMPAIGN

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We consider a differential game of managing the investments in an advertising campaign for the case of n symmetric players. The problem is solved in the class of positional strategies both for the cooperative statement, where the players agree on using controls that maximize the total payoff before the game starts, and for the noncooperative statement, in which the Nash equilibrium is used as a solution. It is shown that the solution of the problem is not unique in both cases. One candidate function is found by means of a detailed analysis. Then the solution is chosen with the use of an economic criterion described by Bass, Krishnamoorthy, Prasad, and Sethi in 2005. The solutions chosen earlier are in complete agreement with the choice with respect to the economic criterion.

Keywords: non-zero-sum differential games, Nash equiiibrium, advertising model.

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