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GAPS IN THE SPECTRUM OF THE LAPLACIAN IN A BAND WITH PERIODIC DELTA INTERACTION

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We consider the Laplace operator in an infinite planar strip with a periodic delta interaction. The width of the strip is fixed and for simplicity is chosen equal to π. The delta interaction is introduced on a periodic system of curves. Each curve consists of a finite number of segments, each having smoothness C^1. The curves are supposed to be strictly internal and do not intersect the boundaries of the strip. The period of their location is $2\varepsilon\pi$, where ε is a sufficiently small number. The function describing the delta interaction is also periodic on the system of curves and is assumed to be bounded and measurable. The main result is the following fact. If $\varepsilon \leqslant \varepsilon_0$, where ε_0 is a certain explicitly calculated number and the norm of the function describing the delta interaction is smaller than some explicit constant, then a lower part of the spectrum of the operator has no internal gaps. The lower part is understood as the band of the spectrum until some point, which is explicitly calculated in terms of the parameter ε as a rather simple function. This result can be considered as a first step to the proof of the strengthened Bethe–Sommerfeld conjecture on the complete absence of gaps in the spectrum of the operator for a sufficiently small period of location of delta interactions.

Keywords: periodic operator, Laplacian, delta interaction, band spectrum, absence of gaps.

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