TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 24 No. 2 **2018**

MSC: 05C25

DOI: 10.21538/0134-4889-2018-24-2-34-39

CODES IN SHILLA DISTANCE-REGULAR GRAPHS

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Let Γ be a distance-regular graph of diameter 3 containing a maximal 1-code C, which is locally regular and perfect with respect to the last neighborhood. Then Γ has intersection array $\{a(p+1), cp, a+1; 1, c, ap\}$ or $\{a(p+1), (a+1)p, c; 1, c, ap\}$, where $a = a_3$, $c = c_2$, and $p = p^3_{33}$ (Jurišić, Vidali). In the first case, Γ has eigenvalue $\theta_2 = -1$ and the graph Γ_3 is pseudogeometric for $GQ(p+1, a)$. In the second case, Γ is a Shilla graph. We study Shilla graphs in which every two vertices at distance 2 belong to a maximal 1-code. It is proved that, in the case $\theta_2 = -1$, a graph with the specified property is either the Hamming graph $H(3, 3)$ or a Johnson graph. We find necessary conditions for the existence of Q-polynomial Shilla graphs in which any two vertices at distance 3 lie in a maximal 1-code. In particular, we find two infinite families of feasible intersection arrays of Q-polynomial graphs with the specified property: $\{b(b^2-3b)/2, (b-2)(b-1)^2/2, (b-2)t/2; 1, bt/2, (b^2-3b)(b-1)/2\}$ (graphs with $p^3_{33} = 0$) and $\{b^2(b-4)/2, (b^2-4b+2)(b-1)/2, (b-2)l/2; 1, bl/2, (b^2-4b)(b-1)/2\}$ (graphs with $p^3_{33} = 1$).

Keywords: distance-regular graph, graph automorphism.

REFERENCES

1. Brouwer A.E., Cohen A.M., Neumaier A. *Distance-Regular Graphs*. Berlin; Heidelberg; N Y: Springer-Verlag, 1989, 495 p. ISBN: 0387506195 .
2. Koolen J.H., Park J. Shilla distance-regular graphs. *Europ. J. Comb.*, 2010, vol. 31, no. 8, pp. 2064–2073. doi: 10.1016/j.ejc.2010.05.012 .
3. Vidali J. *Kode v razdaljno regularnih grafih*, Doctorska dissertacija, Univerza v Ljubljani, Ljubljana, 2013, 155 str.
4. Jurisic A., Vidali J. Extremal 1-codes in distance-regular graphs of diameter 3. *Des. Codes Cryptogr.*, 2012, vol. 65, no. 1-2, pp. 29–47. doi: 10.1007/s10623-012-9651-0 .
5. Makhnev A.A., Nirova M.S. Shilla distance-regular graphs with $b_2 = c_2$. *Mat. Zametki*, 2018, vol. 103, no. 5, pp. 730–744 (in Russian). doi: 10.4213/mzm11503 .
6. Makhnev A.A., Belousov I.N. To the theory of Shilla graphs with $b_2 = c_2$. *Sib. Elektron. Mat. Izv.*, 2017, vol. 14, pp. 1135–1146 (in Russian). doi: 10.17377/semi.2017.14.097 .
7. Koolen J.H., Park J., Yu H. An inequality involving the second largest and smallest eigenvalue of a distance-regular graph. *Linear Algebra Appl.*, 2011, vol. 434, no. 12, pp. 2404–2412. doi: 10.1016/j.laa.2010.12.032 .

The paper was received by the Editorial Office on Dezember 25, 2017.

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Cite this article as:
I. N. Belousov. Codes in Shilla distance-regular graphs, *Trudy Inst. Mat. Mekh. UrO RAN*, 2018, vol. 24, no. 2, pp. 34–39 .