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ON THE DEFINITION OF A BROWNIAN SHEET

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We study the properties of a Brownian sheet, which is a random field generalizing the Brownian motion. It is demonstrated that different sets of properties can be used to define this random function, just as in the case of the Brownian motion. We formulate four definitions of the Brownian motion and, based on them, four definitions of a Brownian sheet. An interesting property of the Brownian motion, which is important for our discussion, is the fact that a process with continuous trajectories and independent increments starting from zero is Gaussian (J. Doob's theorem). In the present paper, we generalize this statement to the case of random fields, which allows us to prove the equivalence of the formulated definitions of a Brownian sheet.

Keywords: Brownian sheet, Brownian motion, Wiener process, Gaussian process, Wiener–Chentsov random field.

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