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## ON INTEGRAL LEBESGUE CONSTANTS OF LOCAL SPLINES WITH UNIFORM KNOTS

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We study the stability properties of generalized local splines of the form

$$S(x) = S(f, x) = \sum_{j \in \mathbb{Z}} y_j B_\varphi \left( x + \frac{3h}{2} - jh \right) \quad (x \in \mathbb{R}),$$

where  $\varphi \in C^1[-h, h]$  for  $h > 0$ ,  $\varphi(0) = \varphi'(0) = 0$ ,  $\varphi(-x) = \varphi(x)$  for  $x \in [0; h]$ ,  $\varphi(x)$  is nondecreasing on  $[0; h]$ ,  $f$  is an arbitrary function from  $\mathbb{R}$  to  $\mathbb{R}$ ,  $y_j = f(jh)$  for  $j \in \mathbb{Z}$ , and

$$B_\varphi(x) = m(h) \begin{cases} \varphi(x), & x \in [0; h], \\ 2\varphi(h) - \varphi(x-h) - \varphi(2h-x), & x \in [h; 2h], \\ \varphi(3h-x), & x \in [2h; 3h], \\ 0, & x \notin [0; 3h] \end{cases}$$

with  $m(h) > 0$ . Such splines were constructed by the author earlier. In the present paper we calculate the exact values of their integral Lebesgue constants (the norms of linear operators from  $l$  to  $L$ ) on the axis  $\mathbb{R}$  and on any segment of the axis for a certain choice of the boundary conditions and the normalizing factor  $m(h)$  of the spline  $S$ .

Keywords: Lebesgue constants, local splines, boundary conditions.

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