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# ASYMPTOTIC EXPANSION OF A SOLUTION TO A SINGULARLY PERTURBED OPTIMAL CONTROL PROBLEM WITH A CONVEX INTEGRAL PERFORMANCE INDEX WHOSE TERMINAL PART DEPENDS ON SLOW VARIABLES ONLY

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We consider an optimal control problem with a convex integral performance index for a linear system with fast and slow variables in the class of piecewise continuous controls with smooth constraints on the control

$$\begin{cases} \dot{x}_\varepsilon = A_{11}x_\varepsilon + A_{12}y_\varepsilon + B_1 u, & t\in[0,T], \quad \|u\|\leqslant 1,\\ \varepsilon\dot{y}_\varepsilon = A_{22}y_\varepsilon + B_2 u, & x_\varepsilon(0)=x^0, \quad y_\varepsilon(0)=y^0,\\ J(u):=\varphi(x_\varepsilon(T)) + \int\limits_0^T \|u(t)\|^2\,dt \to \min, & \end{cases}$$

where $x\in\mathbb{R}^n$, $y\in\mathbb{R}^m$, $u\in\mathbb{R}^r$, $A_{ij}$ and $B_i$ for $i,j=1,2$ are constant matrices of corresponding dimension, and the function $\varphi(\cdot)$ is continuously differentiable in $\mathbb{R}^n$, strictly convex, and cofinite in the sense of convex analysis. In the general case, Pontryagin's maximum principle is applied as a necessary and sufficient optimality condition in this problem, and there exists a unique vector $l_\varepsilon$ that defines an optimal control by the formula

$$u_\varepsilon(T-t) = \frac{C_\varepsilon^*(t)l_\varepsilon}{S\left(\|C_\varepsilon^*(t)l_\varepsilon\|\right)},$$

where

$$C_\varepsilon(t) := e^{\mathcal{A}_\varepsilon t}\mathcal{B}_\varepsilon = e^{A_{11}t}B_1 + \varepsilon^{-1}\mathcal{W}_\varepsilon(t)B_2, \qquad S(\xi) := \begin{cases} 2, & 0\leqslant\xi\leqslant 2,\\ \xi, & \xi>2. \end{cases}$$

The main difference of this problem from the author's previous papers is that the terminal part of the performance index depends on the slow variables only and the control system has a more general form. It is proved that, in the case of a finite number of points where the type of the control is changed, a power asymptotic expansion can be constructed for the initial vector $l_\varepsilon$ of the conjugate system that defines the type of the optimal control.

Keywords: optimal control, singularly perturbed problems, asymptotic expansion, small parameter.

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