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## ADAPTIVE MIRROR DESCENT ALGORITHMS IN CONVEX PROGRAMMING PROBLEMS WITH LIPSCHITZ CONSTRAINTS

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The paper is devoted to new modifications of recently proposed adaptive Mirror Descent methods for convex minimization problems in the case of several convex functional constraints. Methods for problems of two types are considered. In problems of the first type, the objective functional is Lipschitz (generally, nonsmooth). In problems of the second type, the gradient of the objective functional is Lipschitz. We also consider the case of a nonsmooth objective functional equal to the maximum of smooth functionals with Lipschitz gradient. In all the cases, the functional constraints are assumed to be Lipschitz and, generally, nonsmooth. The proposed modifications make it possible to reduce the running time of the algorithm due to skipping some of the functional constraints at nonproductive steps. We derive bounds for the convergence rate, which show that the methods under consideration are optimal from the viewpoint of lower oracle estimates. The results of numerical experiments illustrating the advantages of the proposed procedure for some examples are presented.

Keywords: adaptive Mirror Descent, Lipschitz functional, Lipschitz gradient, productive step, nonproductive step.

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