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ON HILBERT–POINCARE SERIES OF ASSOCIATIVE NILALGEBRAS GENERATED BY TWO NILELEMENTS

A. I. Sozutov, G. P. Egorychev, I. O. Aleksandrova

The coefficients of the Hilbert–Poincare series $H_A(t) = \sum_{k=0}^{\infty} a_k t^k$ of a graded associative algebra $A = \langle\langle x, y | x^m, y^n\rangle\rangle$ with unit are calculated (Theorems 1 and 2). There are no other constraints on the algebra. The problem is the combinatorial search for compact formulas (and asymptotics) for the number of associative words of fixed length in the alphabet $\{x, y\}$ not containing the subwords x^m and y^n. Working with recurrence relations, generating functions, and combinatorial sums, we use operations with power series (of one variable) and elements of the theory of residues for complex variables. These methods supplement the Golod–Shafarevich theorem, which is inapplicable for $d = 2$ and $m, n \leq 9$. In connection with Aleshin, Grigorchuk, and Gupta groups, we pay a special attention to the small values $m, n \leq 4$. An asymptotic expansion of the coefficients a_k is found, and a_k are compared with the coefficients of the series $\sum_{k=0}^{\infty} c_k t^k$, which is the inverse of the polynomial $1-2t+t^m+t^n$. We also consider the cases of negative coefficients c_k and inequalities $c_k > a_k$, which are excluded by the conditions of the Golod–Shafarevich theorem. However, additional relations sufficient for obtaining infinite-dimensional nilalgebras cannot be found yet because the obtained formulas are rather cumbersome.

Keywords: associative nilalgebra, Hilbert–Poincare series.

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Anatolii Ilich Sozutov, Dr. Phys.-Math. Sci., Prof., Siberian Federal University, Krasnoyarsk, 660041 Russia, e-mail: sozutov_ai@mail.ru .

Georgii Petrovich Egorychev, Dr. Phys.-Math. Sci., Prof., Siberian Federal University, Krasnoyarsk, 660041 Russia, e-mail: gegorych@mail.ru .

Inna Olegovna Aleksandrova, Siberian Federal University, Krasnoyarsk, 660041 Russia, e-mail: aio40@mail.ru .

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