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STUDY OF TEST VOLTERRA EQUATIONS OF THE FIRST KIND IN INTEGRAL MODELS OF DEVELOPING SYSTEMS

A. S. Apartsyn, I. V. Sidler

Volterra equations of the first kind are an important element of integral models of developing systems. They describe the balance between the required level of system development and the possibility of achieving this level by a set of elements of the system belonging to different age groups. The solution to the balance equation, which is continuous on any finite time interval, inevitably becomes unstable over time for some relations between the efficiency coefficients of the elements (kernels of the corresponding operators). The simplest test equations allow us to understand the specifics of this phenomenon. Such equations were introduced earlier for the case of two age groups, and we generalize them to the case of three age groups of elements and investigate the obtained equations. The main theoretical result, formulated in Theorem 2, is a majorant estimate for a grid node of any quadrature method for the numerical solution of a test equation where the error of the grid solution exceeds for the first time a given arbitrarily large threshold in the case of using a computer with a fixed rounding error. The result is illustrated by calculations for model examples with the use of modified methods of left and middle rectangle. The developed technique can be naturally extended to the case of an arbitrary number of age groups.

Keywords: developing system, three age groups, test Volterra equation of the first kind, numerical solution, instability.

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Anatoliy Solomonovich Apartsyn, Dr. Phys.-Math. Sci., Melentiev Energy Systems Institute Siberian Branch of the Russian Academy of Sciences, Irkutsk, 664033 Russia, e-mail: apartsyn@isem.irk.ru.

Inna Vladimirovna Sidler, PhD, Melentiev Energy Systems Institute Siberian Branch of the Russian Academy of Sciences, Irkutsk, 664033 Russia, e-mail: inna.sidler@mail.ru.

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