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ON THE VERTEX ADJACENCY IN A POLYTOPE OF CONNECTED k-FACTORS

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Combinatorial characteristics of polytopes associated with combinatorial optimization problems can be considered to some extent as the intractability characteristics of these problems. For example, the NP-completeness of verifying the nonadjacency of vertices in the polytope of a problem quite often accompanies the NP-hardness of the problem. Another important characteristic of the polytope graph of a problem is its clique number. For a rather wide class of algorithms, the clique number is a lower bound for the time complexity of the problem. In addition, for the clique number of polytope graphs, there are known exponential lower bounds for a large number of intractable problems and known polynomial upper and lower bounds for problems solvable in polynomial time. In the present paper we consider the polytope of the problem on a weighted connected spanning k-regular subgraph (a connected k-factor) of a complete n-vertex graph; for $k = 2$, this is the polytope of the symmetric traveling salesman problem. For the values of k satisfying the conditions $k \geq 3$ and $\lceil k/2 \rceil \leq n/8 - 1$, we show that the problem of verifying the nonadjacency of vertices of this polytope is NP-complete and the clique number is exponential in n. The proofs are based on the reduction to the case $k = 2$.

Keywords: k-factor, polytope, adjacency of vertices, clique number of a graph.

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