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ON DISTANCE-REGULAR GRAPHS WITH $\theta_2 = -1$

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Let a distance-regular graph Γ of diameter 3 have eigenvalue $\theta_2 = -1$. Then $\Delta = \bar{\Gamma}_3$ is a pseudo-geometric graph for $pG_{c_3}(k, b_1/c_2)$ containing v Delsarte cliques $u^{\perp}$ of order $k+1$. In the case $a_1 = 0$ we have a partition of the subgraph $\Delta(u)$ by cliques $w^{\perp} - \{u\}$, where $w \in \Gamma(u)$. If there exists a strongly regular graph with parameters (176,49,12,14) in which neighborhoods of vertices are 7×7-lattices, then there exists a distance-regular graph with intersection array $\{7, 6, 6; 1, 1, 2\}$. If Δ contains an n-coclique $\{u, u_2, ..., u_n\}$, then there are $k_3 - (n-1)(a_3+1)$ vertices in $\Gamma_3(u) - \cup_{i=2}^{n}\Gamma(u_i)$, which yields a new upper bound for the order of a clique in Γ_3. Moreover, it is proved that distance-regular graphs with intersection arrays $\{44, 35, 3; 1, 5, 42\}$ and $\{27, 20, 7; 1, 4, 21\}$ do not exist.

Keywords: distance-regular graph, eigenvalue, strongly regular graph.

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