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ON THE COINCIDENCE OF THE MINIMAX SOLUTION AND THE VALUE FUNCTION IN A TIME-OPTIMAL GAME WITH A LIFELINE

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We consider time-optimal differential games with a lifeline. In such games, as usual, there is a terminal set to which the first player tries to guide the system as fast as possible, and there is also a set, called a lifeline, such that the second player wins when the system attains this set. The payoff is the result of applying Kruzhkov's change to the time when the system reaches the terminal set. We also consider Hamilton–Jacobi equations corresponding to such games. The existence of a minimax solution of a boundary value problem for a Hamilton–Jacobi type equation is proved. For this we introduce certain strong assumptions on the dynamics of the game near the boundary of the game domain. More exactly, the first and second players can direct the motion of the system to the terminal set and the lifeline, respectively, if the system is near the corresponding set. Under these assumptions, the value function is continuous in the game domain. The coincidence of the value function and the minimax solution of the boundary value problem is proved under the same assumptions.

Keywords: time-optimal differential games with a lifeline, value function, Hamilton–Jacobi equations, minimax solution.

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