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ASYMPTOTIC CONFIDENCE INTERVAL FOR A DISCONTINUITY POINT
OF A PROBABILITY DENSITY FUNCTION

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We consider the problem of interval estimation of an unknown parameter $\theta \in \Theta \subset R$ of a distribution density $f(x, \theta)$ (with respect to the Lebesgue measure) for a sample X_1, \dots, X_n of large size. It is assumed that the density has a discontinuity of the first kind at the point $x = \theta$. We construct a confidence interval based on a known maximum likelihood estimator θ_n^* and the distribution function $G(x, \theta)$ found by the authors earlier, which is the limit of the sequence of distribution functions of normalized maximum likelihood estimators $n(\theta_n^* - \theta)$. It is proved that the resulting confidence interval is asymptotically exact. We also describe a method for the “fast” calculation of maximum likelihood estimators for a discontinuity point of a density.

Keywords: estimation of a discontinuity point of a probability density, maximum likelihood estimators, asymptotic confidence interval, limiting distributions of statistical estimators.

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