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THE STUDY OF EQUATIONS FOR PROBABILITY CHARACTERISTICS OF RANDOM PROCESSES DESCRIBED BY STOCHASTIC EQUATIONS

I. V. Melnikova, D. I. Smetannikov

The paper is devoted to the comparison of two approaches to investigating the relationship between processes with a given set of properties determined by properties of solutions to stochastic equations with Wiener-type randomness and partial differential equations for probabilistic characteristics of these processes, including equations for densities of transition probabilities. The first approach is based on the application of the Ito formula for diffusion processes, which are solutions of stochastic equations, whereas the second approach employs the continuity properties of the process and the existence of limits characterizing the local behavior of solutions to stochastic equations. In the course of the comparison, the following is established. In the first approach, in the proof of the relationship between the coefficients of the stochastic equation and the coefficients of the corresponding partial differential equation, the key role is played by the Markov and martingale properties of functions of solutions to stochastic equations. The second approach is based on the existence of global moments of the first and second order for solutions of stochastic Cauchy problems, which in the case of stochastic equations with Wiener-type randomness define their local behavior. As an application, we model a stochastic problem for a specific system using the connection with equations for the transition probabilities of the process determined by statistical data.

Keywords: Wiener process, Markov process, martingale, Ito formula, Kolmogorov equation, probability characteristics.

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Irina Valeryanovna Melnikova, Dr. Phys.-Math. Sci., Prof., Institute of Natural Sciences and Mathematics Ural Federal University, Yekaterinburg, 620002 Russia, e-mail: irina.melnikova@urfu.ru .

Daniil Il'ich Smetannikov, graduate student, Institute of Natural Sciences and Mathematics Ural Federal University, Yekaterinburg, 620002 Russia, e-mail: smetannikovdi@yandex.ru .

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