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CAMERON–LIEBLER LINE CLASSES IN PG$(N, 5)$

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A Cameron–Liebler line class with parameter x in a finite projective geometry PG(n, q) of dimension n over a field with q elements is a set $\mathcal{L}$ of lines such that any line ℓ intersects $x(q+1) + \chi_{\mathcal{L}}(\ell)(q^{n-1} + \cdots + q^2 - 1)$ lines from $\mathcal{L}$, where $\chi_{\mathcal{L}}$ is the characteristic function of the set $\mathcal{L}$. The generalized Cameron–Liebler conjecture states that for $n > 3$ all Cameron–Liebler classes are known and have a trivial structure in some sense (more exactly, up to complement, the empty set, a point-pencil, all lines of a hyperplane, and the union of the last two for nonincident point and hyperplane). The validity of the conjecture was proved earlier by other authors for the cases $q = 2$, 3, and 4. In the present paper we describe an approach to proving the conjecture for given q under the assumption that all Cameron–Liebler classes in PG$(3, q)$ are known. We use this approach to prove the generalized Cameron–Liebler conjecture in the case $q = 5$.

Keywords: finite projective geometry, Cameron–Liebler line classes.

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