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## PRODUCTS AND JOINS OF LOCALLY NORMAL FITTING CLASSES

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Let  $\pi$  be a nonempty set of primes. A nontrivial Fitting class  $\mathfrak{F}$  is said to be normal in the class  $\mathfrak{S}_\pi$  of all finite soluble  $\pi$ -groups or  $\pi$ -normal (we write  $\mathfrak{F} \triangleleft \mathfrak{S}_\pi$ ) if  $\mathfrak{F} \subseteq \mathfrak{S}_\pi$  and the  $\mathfrak{F}$ -radical of every  $\pi$ -group  $G$  is a  $\mathfrak{F}$ -maximal subgroup of  $G$ . If  $\pi$  is the set of all primes, then  $\mathfrak{F}$  is called normal. The product  $\mathfrak{F}\mathfrak{H}$  of Fitting classes  $\mathfrak{F}$  and  $\mathfrak{H}$  is called  $\pi$ -normal if  $\mathfrak{F}\mathfrak{H}$  is a  $\pi$ -normal Fitting class. We prove the existence of  $\pi$ -normal products of Fitting classes factorizable by non- $\pi$ -normal factors. Assume that  $\mathbb{P}$  is the set of all primes,  $\emptyset \neq \pi \subseteq \mathbb{P}$ ,  $\mathfrak{F}$  is some Fitting class of  $\pi$ -groups, and  $\omega = \sigma(\mathfrak{F})$  is the set of all prime divisors of all groups from  $\mathfrak{F}$ . It is proved that if  $\mathfrak{F}^2 = \mathfrak{F}$  and  $\mathfrak{H}$  is the class of all  $\pi$ -groups with central  $\omega$ -socle, then the product  $\mathfrak{F}\mathfrak{H}$  is  $\pi$ -normal although each of the factors  $\mathfrak{F}$  and  $\mathfrak{H}$  is not  $\pi$ -normal. The lattice join  $\mathfrak{F} \vee \mathfrak{H}$  of Fitting classes  $\mathfrak{F}$  and  $\mathfrak{H}$  is the Fitting class generated by  $\mathfrak{F} \cup \mathfrak{H}$ . If  $\mathfrak{F} \vee \mathfrak{H}$  is a  $\pi$ -normal Fitting class, then  $\mathfrak{F} \vee \mathfrak{H}$  is called  $\pi$ -normal. Let  $\mathfrak{F}$  and  $\mathfrak{H}$  be Fitting classes of  $\pi$ -groups. We prove that the lattice join  $\mathfrak{F} \vee \mathfrak{H}$  is a  $\pi$ -normal Fitting class if and only if  $\mathfrak{F}$  or  $\mathfrak{H}$  is a  $\pi$ -normal Fitting class.

Keywords:  $\mathfrak{F}$ -radical, Fitting class,  $\pi$ -normal Fitting class, join of Fitting classes.

## REFERENCES

1. Doerk K., Hawkes T. *Finite soluble groups*. Berlin; New York: Walter de Gruyter & Co, 1992, Ser.: De Gruyter Expo. Math., 4, 891 p. ISBN: 978-3-11-087013-8.
2. Blessohl D., Gaschütz W. Über normale Schunk- und Fittingklassen. *Math. Z.*, 1970, vol. 118, no. 1, pp. 1–8. doi: 10.1007/BF01109888.
3. Vorob'ev N.T., Martsinkevich A.V. Finite  $\pi$ -groups with normal injectors. *Sib. Math. J.*, 2015, vol. 56, no. 4, pp. 624–630. doi: 10.17377/smzh.2015.56.406.
4. Cossey J. Products of Fitting classes. *Math. Z.*, 1975, vol. 141, no. 3, pp. 289–295. doi: 10.1007/BF01247314.
5. *Kourovka notebook: Unsolved problems of group theory*. 11th ed., IM RAN, Novosibirsk, 1990, 126 p. (in Russian).
6. Vorob'ev N.T. On the factorization of local and non-local products of finite groups of non-local formations. *Proc. 7th Reg. Sci. Sess. Math., Sect. Algebra and Number Theory*. Kalsk, 1990, pp. 9–13.
7. Vedernikov V.A. Local formations of finite groups. *Math. Notes*, 1989, vol. 46, no. 6, pp. 910–913. doi: 10.1007/BF01158624.
8. Vorob'ev N.T., Skiba A.N. Local products of non-local Fitting classes. *Voprosy algebr*, 1995, no. 8, pp. 55–58 (in Russian).
9. Shpakov V.V., Vorobyev N.T. Local factorisations of nonlocal Fitting classes. *Discrete Math. Appl.*, 2008, vol. 18, no. 4, pp. 439–446. doi: 10.1515/DMA.2008.032.
10. Lockett F.P. The Fitting class  $\mathfrak{F}^*$ . *Math. Z.*, 1974, vol. 137, no. 2, pp. 131–136. doi: 10.1007/BF01214854.
11. Cusack E. The join of two Fitting classes. *Math. Z.*, 1979, vol. 167, no. 1, pp. 37–47. doi: 10.1007/BF01215242.
12. Beidleman J.C. On products and normal Fitting classes. *Arch. Math.*, 1977, vol. 28, no. 1, pp. 347–356. doi: 10.1007/BF01223934.
13. Gaschütz W. Lectures of subgroups of Sylow type in finite soluble groups. *Notes on pure mathematics*, 1979, no. 11, pp. 1–100.
14. Savelyeva N.V., Vorob'ev N.T. Maximal on strong  $\pi$ -containment Fitting classes. *Izv. Gomel. Gos. Univ. im. F. Skoriny*, 2008, no. 2(47), pp. 157–168 (in Russian).

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