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ON COMPUTING A CLASS OF INTEGRALS OF RATIONAL FUNCTIONS WITH PARAMETERS AND SINGULARITIES ON COMPLEX HYPERPLANES

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We give an algorithm for computing the integral

$$\int_{|\xi_1|=1} \dots \int_{|\xi_n|=1} \frac{f(\xi)}{\prod\limits_{j=1}^{m} (a_{j,1}z_1\xi_1 + \dots + a_{j,n}z_n\xi_n + c_j)^{t_j}} \cdot \frac{d\xi_1}{\xi_1} \dots \frac{d\xi_n}{\xi_n},$$

where the integration set is the distinguished boundary of the unit polydisk in $\mathbb{C}^n$, the function $f(\xi)$ is holomorphic in a neighborhood of this set, and $\prod_{j=1}^{m}(a_{j,1}z_1\xi_1+\dots+a_{j,n}z_n\xi_n+c_j) \neq 0$ for points $z = (z_1, \dots, z_n)$ of a connected n-circular set $G \subset \mathbb{C}^n$. For points of the distinguished boundary, whose coordinates satisfy the relations $|\xi_1| = 1$, …, $|\xi_n| = 1$, the sets $\{V_j\} = \{(z_1, \dots, z_n) \in \mathbb{C}^n \colon a_{j,1}z_1\xi_1 + \dots + a_{j,n}z_n\xi_n + c_j = 0\}$ are n-circular, and it is convenient to study their mutual arrangement in $\mathbb{C}^n$ by using the projection $\pi \colon \mathbb{C}^n \to \mathbb{R}^n_+$, where $\pi(z_1, \dots, z_n) = (|z_1|, \dots, |z_n|)$. Each connected set $\pi(\{V_j\})$ divides $\mathbb{R}^n_+$ into at most $n + 1$ disjoint nonempty parts, and $\pi(G)$ belongs to one of them. Therefore the number of variants of the mutual arrangement of the sets G and $\{V_1\}, \dots, \{V_m\}$ in $\mathbb{C}^n$, which influences the value of the integral, does not exceed $(n + 1)^m$. In Theorems 1 and 2 we compute the integral for two of these variants. An example of computing a double integral by applying its parameterization and one of the theorems is given.

Keywords: integral representation, n-circular domain, complex plane.

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