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ON THE CONVERGENCE OF SOLUTIONS OF VARIATIONAL PROBLEMS WITH IMPLICIT CONST-RAINTS DEFINED BY RAPIDLY OSCILLATING FUNCTIONS

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For functionals defined on variable Sobolev spaces, we establish a series of results on the convergence of their minimizers and minimum values on sets of functions subject to implicit constraints by means of periodic rapidly oscillating functions. In connection with the formulation and justification of these results, we introduce the definition of Γ-convergence of functionals corresponding to the given sets of constraints. The specificity of the introduced definition is that it refers to the convergence of a sequence of functionals defined on variable Sobolev spaces to a function on the real line. The considered minimization problems have the feature that, to justify the convergence of a sequence of their solutions, the strong connectedness of the domains of definition of the corresponding functionals is not required, while this connectedness is essential, for instance, in the study of the convergence of solutions of the Neumann variational problems and variational problems with explicit unilateral and bilateral constraints in variable domains. In addition to the mentioned results, we establish theorems on the Γ-compactness of sequences of functionals with respect to the given sets of constraints.

Keywords: variational problem, implicit constraint, variable domains, functional, minimizer, minimum value, Γ-convergence.

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