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## ON FIXED POINTS OF MULTIVALUED MAPPINGS IN SPACES WITH A VECTOR-VALUED METRIC

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Nadler's theorem on a fixed point of a multivalued mapping is extended to spaces with a vector-valued metric. A vector-valued metric is understood as a mapping with the properties of a usual metric and values in a linear normed ordered space. We prove an analog of Nadler's theorem and apply it to a system of integral inclusions in a space of summable functions. Then we study a boundary value problem with multivalued conditions for systems of functional differential equations by means of reduction to a system of integral inclusions. Conditions for the existence of solutions are obtained and estimates of the solutions are given. The existence conditions do not contain the convexity requirement for the values of the multivalued function generating a Nemytskii operator.

Keywords: space with a vector-valued metric, contracting multivalued mapping, fixed point, integral inclusion.

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