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ON GENERATORS OF A MATRIX ALGEBRA AND SOME OF ITS SUBALGEBRAS

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It is shown that a full matrix algebra M_n admits a generator system consisting of two nilpotent matrices P and Q such that any matrix $A = (a_{ij})$ is expressed explicitly in terms of P and Q as $A = \sum_{i \neq j} a_{ij} P^{i-1} Q P^{n-j}$, $i, j = 1, 2, \ldots, n$. We show how this representation can be applied to calculate the powers of the coefficient matrix A of a linear system $x_{n+1} = Ax_n + r_n$ modeling heat exchange in a regenerative air preheater. More exactly, we obtain convenient recursive formulas for the elements of A^k, $k = 1, 2, \ldots$. We also consider the problem of constructing a simple system of generators for the subalgebras of diagonal and triangular matrices. We observe that a generating matrix of the subalgebra of diagonal matrices is related to the Lagrange interpolation formula and prove that the subalgebra of triangular matrices is generated by a diagonal matrix with pairwise different elements and first skew diagonal. It is shown that a triangular matrix $A \in T_n$ with pairwise different diagonal elements can be reduced to a Jordan form within the subalgebra T_n; i.e., there exists $L \in T_n$ such that $L^{-1}AL$ is diagonal. In the general case this property does not hold for arbitrary matrices from T_n.

Keywords: matrix algebra, system of generators, nilpotent matrix, matrix unit, subalgebra, Jordan form, interpolation polynomial, discrete system, air preheater, heat exchange.

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