

MSC: 49K15

DOI: 10.21538/0134-4889-2017-24-1-76-90

**VARIATIONS OF THE v -CHANGE OF TIME IN PROBLEMS
WITH STATE CONSTRAINTS****A. V. Dmitruk, N. P. Osmolovskii**

For a general optimal control problem with a state constraint, we propose a proof of the maximum principle based on a v -change of the time variable $t \mapsto \tau$, under which the original time becomes yet another state variable subject to the equation $dt/d\tau = v(\tau)$, while the additional control $v(\tau) \geq 0$ is piecewise constant, and its values are arguments of the new problem. Since the state constraint generates a continuum of inequality constraints in this problem, the necessary optimality conditions involve a measure. Rewriting these conditions in terms of the original problem, we get a nonempty compact set of collections of Lagrange multipliers that fulfil the maximum principle on a finite set of values of the control and time variables corresponding to the v -change. The compact sets generated by all possible piecewise constant v -changes are partially ordered by inclusion, thus forming a centered family. Taking any element of their intersection, we obtain a universal optimality condition, in which the maximum principle holds for all values of the control and time.

Keywords: Pontryagin maximum principle, v -change of time, state constraint, semi-infinite problem, Lagrange multipliers, Lebesgue–Stieltjes measure, function of bounded variation, finite-valued maximum condition, centered family of compact sets.

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The paper was received by the Editorial Office on July 26, 2017.

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Cite this article as:

A. V. Dmitruk, N. P. Osmolovskii. Variations of the v -change of time in problems with state constraints, *Trudy Inst. Mat. Mekh. UrO RAN*, 2018, vol. 24, no. 1, pp. 76–92.