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ON THE GEOMETRY OF REACHABLE SETS FOR CONTROL SYSTEMS WITH ISOPERIMETRIC CONSTRAINTS

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A nonlinear control system linear in control variables is considered. The control and the trajectory are subject to a system of isoperimetric constraints in the form of inequalities for integral functionals. We describe the boundary of the reachable set of the system at a given time and show that an admissible control taking the system to the boundary of the admissible set is a weakly efficient solution of a certain optimal control problem with a vector criterion if the linearized system is completely controllable. The components of the criterion are integral functionals that specify isoperimetric constraints. The stated result generalizes the authors' earlier results to the case of several consistent integral constraints. The proof is based on the Graves theorem on covering mappings and on the properties of the derivative of the "input–output" mapping and of the constraints. The result remains valid if the initial state of the system is not fixed but belongs to a given set. The problem is reduced to a control problem with a scalar criterion depending on parameters. The Chebyshev convolution of integral functionals is chosen as the scalar criterion. Necessary conditions are obtained for the optimality of controls taking the system to the boundary of the reachable set in the form of Pontryagin's maximum principle.

Keywords: control system, isoperimetric constraints, reachable set, maximum principle.

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