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**APPROXIMATION OF MINIMAX SOLUTIONS TO HAMILTON–JACOBI
FUNCTIONAL EQUATIONS FOR DELAY SYSTEMS****M. I. Gomoyunov, N. Yu. Lukoyanov, A. R. Plaksin**

A minimax solution of the Cauchy problem for a functional Hamilton–Jacobi equation with coinvariant derivatives and a condition at the right end is considered. Hamilton–Jacobi equations of this type arise in dynamical optimization problems for time-delay systems. Their approximation is associated with additional questions of the correct transition from the infinite-dimensional functional argument of the desired solution to the finite-dimensional one. Earlier, the schemes based on the piecewise linear approximation of the functional argument and the correctness properties of minimax solutions were studied. In this paper, a scheme for the approximation of Hamilton–Jacobi functional equations with coinvariant derivatives by ordinary Hamilton–Jacobi equations with partial derivatives is proposed and justified. The scheme is based on the approximation of the characteristic functional–differential inclusions used in the definition of the desired minimax solution by ordinary differential inclusions.

Keywords: Hamilton–Jacobi equations, generalized solutions, coinvariant derivatives, finite-dimensional approximations, time-delay systems.

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