ON EQUATIONS OF THE PROGRAM ITERATION METHOD

S. V. Chistyakov

The program iteration method, which is inseparably associated with the name of A. G. Chentsov, first appeared in the study of the so-called zero-sum differential games. At early stages, only one of the two possible dual iterative procedures was considered — the maximin procedure. This can be explained by the special interest of the researchers in the so-called program maximin function, which is conveniently interpreted in geometric terms in games of pursuit. Nevertheless, the dual minimax iterative procedure is of no less interest. The program iteration method is mainly significant because it may be used as the basis for the development of a differential game theory in a closed compact form, which was shown earlier for a version of the method based on a certain modification of iterative operators. The key role in this theory belongs to the theorem that states the existence and uniqueness of a solution of the equation induced by a pair of such operators. In this case, the maximin iterative procedure is used to describe ε-optimal (in some cases, optimal) positional strategies of the first player, while the minimax procedure is used to describe ε-optimal (in some cases, optimal) positional strategies of the second player. This paper investigates the structure of the solution set of the generalized Isaacs–Bellman equation obtained with the use of historically first (not modified) operators of the program iteration method. A theorem that states the existence and uniqueness of the solution to this equation meeting a natural boundary condition is proved under certain assumptions. Thus, it is shown that the original version of the program iteration method can also be used in designing a closed-form differential game theory. However, here we use the so-called recursive strategies rather than positional ones. Such strategies, together with the program iteration method, play an essential role in the analysis of coalition-free differential games.

Keywords: zero-sum differential game, terminal payoff, program iteration method, generalized Isaacs–Bellman equation.

REFERENCES

1. Chentsov A.G. On the structure of a game problem of convergence. *Soviet Math. Dokl.*, 1976 (1975), vol. 16, no. 5, pp. 1404–1408.

2. Chentsov A.G. On a game problem of converging at a given instant of time. *Math. USSR-Sb.*, 1976, vol. 28, no. 3, pp. 353–376. doi: 10.1070/SM1976v028n03ABEH001657 .

3. Chistyakov S.V., Petrosyan L.A. On an approach to the solution of pursuit games. *Vestnik Leningrad. Univ. Math.*, 1977, no. 1, pp. 77–82 (in Russian).

4. Chistyakov S.V. On solving pursuit game problem. *J. Appl. Math. Mech.*, 1979, vol. 41, no. 5, pp. 845–852. doi: 10.1016/0021-8928(77)90167-8 .

5. Krasovskii N.N. *Igrovye zadachi o vstreche dvizhenii.* [Game problems on the encounter of motions]. Moscow, Nauka Publ., 1970, 420 p.

6. Chistyakov S., Nikitin F. On value operators in differential games. *Appl. Math. Sci.*, 2015, vol. 9, no. 59, pp. 2941–2952.

7. Chistyakov S.V. *Operatory znachenija antagonisticheskih differencial'nyh igr.* [Value operators in antagonistic differential games]. Saint Petersburg: Saint Petersburg State University Publ., 1999, 62 p.

8. Nikitin F.F., Chistyakov S.V. Existence and uniqueness theorem for a generalized Isaacs–Bellman equation. *Diff. Eq.*, 2007, vol. 43, no. 6, pp. 757–766. doi: 10.1134/S0012266107060031 .

9. Chistyakov S.V. On functional equations in games of encounter at a prescribed instant. *J. Appl. Math. Mech.*, 1982, vol. 46, no. 5, pp. 704–706. doi: 10.1016/0021-8928(82)90023-5 .

10. Chentsov A.G., Subbotin A.I. An iterative procedure for constructing minimax and viscosity solutions to the Hamilton–Jacobi equations and its generalization. *Proc. Steklov Inst. Math.*, 1999, vol. 224, pp. 286–309.

11. Chistyakov S.V., Nikitin F.F. On regular differential games of persuit with fixed duration. *Vestnik S.-Petersburg Univ. Ser. 10. Prikl. Mat. Inform. Prots. Upr.*, 2014, no. 4, pp. 17–24 (in Russian).

12. Chistyakov S.V. Programmed iterations and universal ε-optimal stratagies in a positional differential game. *Soviet Math. Dokl.*, 1992, vol. 44, no. 1, pp. 354–357.

13. Nikitin F.F. Viscosity solutions and programmed iteration method for Isaacs–Bellman equation. *Vestnik S.-Petersburg Univ. Ser. 10. Prikl. Mat. Inform. Prots. Upr.*, 2014, no. 2, pp. 84–92 (in Russian).

14. Chistyakov S.V. On coalition-free differential games. *Soviet Math. Dokl.*, 1981, vol. 24, no. 1, pp. 166–169.

Sergey Vladimirovich Chistyakov, Dr. Phys.-Math. Sci., Prof., St. Petersburg State University, St. Petersburg, 199034 Russia, e-mail: svch50@mail.ru .