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**DISCRETE APPROXIMATION OF THE HAMILTON — JACOBI EQUATION  
FOR THE VALUE FUNCTION IN AN OPTIMAL CONTROL PROBLEM  
WITH INFINITE HORIZON****A. L. Bagno, A. M. Taras'ev**

An infinite horizon optimal control problem is considered in which the quality functional contains an index with discount factor under the integral sign. The main feature of the problem is the unbounded index, which allows to analyze economic growth models with linear, power, and logarithmic utility functions. A discrete approximation of the Hamilton — Jacobi equation is explored for constructing the value function of the original problem. The Hölder condition and the sublinear growth condition are derived for the solution of the discrete approximation equation. Uniform convergence of solutions of approximation equations to the value function of the optimal control problem is shown. The obtained results can be used to construct grid approximation methods for the value function of an optimal control problem on an infinite time interval. The proposed methods are effective tools in the modeling of economic growth processes.

Keywords: discrete approximation, optimal control, Hamilton — Jacobi equation, viscosity solution, infinite horizon, value function.

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