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## BITOPOLOGICAL SPACES OF ULTRAFILTERS AND MAXIMAL LINKED SYSTEMS

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Issues of the structure of spaces of ultrafilters and maximal linked systems are studied. We consider a widely understood measurable space (a  $\pi$ -system with zero and one) defined as follows: we fix a nonempty family of subsets of a given set closed under finite intersections and containing the set itself (“one”) and the nonempty set (“zero”). Ultrafilters (maximal filters) and maximal linked systems are constructed on this space. Each of the obtained spaces is equipped with a pair of comparable topologies. The resulting bitopological spaces turn out to be consistent in the following sense: each space of ultrafilters is a subspace of the corresponding space of maximal linked systems. Moreover, the space of maximal linked systems with Wallman-type topology is supercompact and, in particular, compact. Possible variants of the  $\pi$ -systems are lattices, semialgebras and algebras of sets, topologies, and families of closed sets of topological spaces.

Keywords: maximal linked system, topological space, ultrafilter.

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