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ON NECESSARY LIMIT GRADIENTS IN CONTROL PROBLEMS WITH INFINITE HORIZON

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We study necessary optimality conditions in control problems with infinite horizon and an overtaking optimality criterion. Under the assumption that all gradients of the payoff function are bounded, we construct a necessary optimality condition for the adjoint variable in terms of the limit points of the gradients $\frac{\partial J}{\partial x}(\xi, 0; \tilde{u}, T)$ as $\xi \to \tilde{x}(0), T \to \infty$. In the case when the gradient of the payoff function is continuous at infinity along an optimal trajectory (the limit point is unique), this condition supplements the system of the maximum principle to a complete system of relations and defines a unique solution. It is shown that the adjoint variable of this solution can be written explicitly with the use of the (Cauchy type) formula proposed earlier by A. M. Aseev and A. V. Kryazhimskii. It is also shown that the solution automatically satisfies one more condition (on the Hamiltonian) proposed recently by A. O. Belyakov for finding solutions optimal with respect to the overtaking criterion. We note that, in the case of the weaker requirement of the existence of the limit $\frac{\partial J}{\partial x}(\tilde{x}(0), 0; \tilde{u}, T)$ as $T \to \infty$, a Cauchy type formula may be inconsistent with the Hamiltonian maximization condition and, hence, with Pontryagin's maximum principle. The key idea of the proof is the application of the theorem on the convergence of subdifferentials for a sequence of uniformly convergent functions within Halkin's scheme.

Keywords: infinite horizon control problem, necessary conditions, transversality conditions at infinity, Pontryagin maximum principle, convergence of subdifferentials.

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