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METHOD OF LIMITING DIFFERENTIAL INCLUSIONS FOR NONAUTONOMOUS DISCONTINUOUS SYSTEMS WITH DELAY

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Functional–differential equations $\dot{x} = f(t, \phi(\cdot))$ with piecewise continuous right-hand sides are studied. It is assumed that the sets M of discontinuity points of the right-hand sides possess the boundedness property in contrast to being zero-measure sets, as in the case of differential equations without delay. This assumption is made largely because the domain of the function f is infinite-dimensional. Solutions to the equations under consideration are understood in Filippov's sense. The main results are theorems on the asymptotic behavior of solutions formulated with the use of invariantly differentiable Lyapunov functionals with fixed-sign derivatives. Nonautonomous systems are difficult to deal with because ω-limiting sets of their solutions do not possess invariance-type properties, whereas sets of zeros of derivatives of Lyapunov functionals may depend on the variable t and extend beyond the space of variables $\phi(\cdot)$. For discontinuous nonautonomous systems, there arises the issue of constructing the limiting differential equations with the use of shifts $f^\tau(t+\tau, \phi(\cdot))$ of the function f. We introduce the notion of limiting differential inclusion without employing limit passages on sequences of shifts of discontinuous or multivalued mappings. The properties of such inclusions are studied. Invariance-type properties of ω-limiting sets of solutions and analogs of LaSalle's invariance principle are established.

Keywords: limiting functional–differential inclusion, asymptotic behavior of solutions, Lyapunov's functional, invariance principle.

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