

MSC: 49N70, 49N75, 91A23, 91A24

DOI: 10.21538/0134-4889-2018-24-1-209-222

**IMPULSE DIFFERENTIAL GAME WITH A MIXED CONSTRAINT
ON THE CHOICE OF THE CONTROL OF THE FIRST PLAYER****V. I. Ukhobotov, I. V. Izmet'shev**

We consider a linear differential game in which the first player can choose both an impulse control and a control subject to a geometric constraint. The first player can use a prescribed amount of resource to form the impulse control. Portions of this amount can be separated at certain times, thus producing "instantaneous" changes of the state vector and complicating the problem. The control of the second player is subject to a geometric constraint. The vectograms of the players are described by the same ball with different time-dependent radii. The terminal set is a ball with fixed radius. The aim of the first player is to bring the state vector to the terminal set at a given time. The aim of the second player is opposite. Necessary and sufficient conditions for meeting the terminal set at the given time are found, and the corresponding controls of the players guaranteeing the achievement of their goals are constructed. A solution of an example illustrating the theory is given.

Keywords: differential game, control, impulse control, capture.

REFERENCES

1. Krasovskii N.N. *Teoriya upravleniya dvizheniem*. [Theory of motion control]. Moscow, Nauka Publ., 1968, 475 p.
2. Krasovskii N.N. On a problem of tracing. *J. Appl. Math. Mech.*, 1963, vol. 27, no. 1, pp. 363–377.
3. Krasovskii N.N., Repin Yu.M., Tret'yakov V.E. Some game situations in theory of control systems. *Izvestiya Akad. Nauk SSSR. Tekhnicheskaya Kibernetika*, 1965, no. 4, pp. 3–23 (in Russian)
4. Krasovskii N.N., Tret'yakov V.E. On a pursuit problem in the case of restrictions on the impulses of control forces. *Diff. Uravn.*, 1966, vol. 2, pp. 587–599 (in Russian).
5. Pozharitskii G.K. Game problem of impulse encounter with an opponent limited in energy. *J. Appl. Math. Mech.*, 1975, vol. 39, no. 4, pp. 555–565.
6. Subbotina N.N., Subbotin A.I. Alternative for the encounter–evasion differential game with constraints on the momenta of players' controls. *J. Appl. Math. Mech.*, 1975, vol. 39, no. 3, pp. 376–385. doi: 10.1016/0021-8928(75)90002-7.
7. Serov V.P., Chentsov A.G. On a programmed linear game-theoretic guidance problem with constraints on the control force impulse. *Autom. Remote Control*, 1993, vol. 54, no. 5, part 1, pp. 755–768.
8. Kumkov S.I., Patsko V.S. Information sets in the pulse control problem. *Autom. Remote Control*, 1997, vol. 58, no. 7, part 2, pp. 1224–1234.
9. Petrov N.N. A problem of group pursuit in the class of impulse strategies of pursuers. *J. Comput. Syst. Sci. Int.*, 2009, vol. 48, no. 2, pp. 199–205. doi: 10.1134/S106423070902004X.
10. Kotlyachkova E.V. About non-stationary problem of simple pursuit in the class of impulse strategies. *Izv. IMI UdGU*, 2015, no. 1(45), pp. 106–113.
11. Chikrii A.A., Matichin I.I. Linear differential games with impulse control of players. *Proc. Steklov Inst. Math.*, 2005, suppl. 1, pp. S68–S81.
12. Belousov A.A. Differential games under integral constraints with impulse controls. *Dokl. NAN Ukrainy*, 2013, no. 11, pp. 37–42.
13. Tukhtasinov M. A linear differential game of pursuit with impulse and integrally constrained controls of the players. *Tr. Inst. Mat. Mekh.*, 2016, vol. 22, no. 3, pp. 273–282 (in Russian). doi: 10.21538/0134-4889-2016-22-3-273-282.

14. Ukhobotov V.I. A linear differential game with constraints imposed on the control impulses. *J. Appl. Math. Mech.*, 1988, vol. 52, no. 3, pp. 277–283. doi: 10.1016/0021-8928(88)90078-0.
15. Ukhobotov V.I. *Metod odnomernogo proektirovaniya v lineinykh differentsial'nykh igrakh s integral'nymi ogranicheniyami*. [Method of one-dimensional projecting in linear differential games with integral constraints]. Chelyabinsk, Chelyabinsk State University Publ., 2005, 124 p. ISBN: 5-7271-0725-3.
16. Ukhobotov V.I., Zaitseva O.V. A linear problem of pulse encounter at a given time under interference. *Proc. Steklov Inst. Math.*, 2011, vol. 272, suppl. 1, pp. 215–228. doi: 10.1134/S0081543811020167.
17. Aubin J.-P., Seube N. Conditional viability for impulse differential games. *Annals of Operations Research*, 2005, vol. 137, no. 1, pp. 269–297. doi: 10.1109/CDC.2002.1184364.
18. Pontryagin L.S. Linear differential games of pursuit. *Mathematics of the USSR-Sbornik*, 1981, vol. 40, no. 3, pp. 285–303. doi: 10.1070/SM1981v040n03ABEH001815.
19. Ukhobotov V.I. Synthesis of a control in differential games of one type with fixed time. *Vestnik Chelyabinsk. Univ. Ser. 3 Mat. Mekh.*, 1996, no. 1(3), pp. 178–184 (in Russian).
20. Ukhobotov V.I. On a class of linear differential games with impulse controls. *J. Appl. Math. Mech.*, 1974, vol. 38, no. 4, pp. 550–557.
21. Lusternik L.A. Sobolev V.J. *Elements of functional analysis (International monographs on advanced mathematics and physics)*. Delhi, Hindustan Publishing Corp., 1974, 360 p. ISBN: 0470556501. Original Russian text published in Lyusternik L.A., Sobolev V.I. *Elementy funktsional'nogo analiza*. Moscow, Nauka Publ., 1965, 520 p.

The paper was received by the Editorial Office on August 31, 2017.

Viktor Ivanovich Ukhobotov, Dr. Phys.-Math. Sci., Prof., Chelyabinsk State University, Chelyabinsk, 454001 Russia, e-mail: ukh@csu.ru.

Igor' Vyacheslavovich Izmet'ev, Chelyabinsk State University, Chelyabinsk, 454001 Russia, e-mail: j748e8@gmail.com.

Cite this article as:

A. G. Chentsov. Bitopological spaces of ultrafilters and maximal linked systems, *Trudy Inst. Mat. Mekh. UrO RAN*, 2018, vol. 24, no. 1, pp. 209–222.