

MSC: 58C06

DOI: 10.21538/0134-4889-2018-24-1-200-208

SPACE OF CONTINUOUS SET-VALUED MAPPINGS WITH CLOSED UNBOUNDED VALUES

A. A. Tolstonogov

We consider a space of continuous multivalued mappings defined on a locally compact space \mathcal{T} with countable base. Values of these mappings are closed not necessarily bounded sets from a metric space $(X, d(\cdot))$ in which closed balls are compact. The space $(X, d(\cdot))$ is locally compact and separable. Let Y be a dense countable set from X . The distance $\rho(A, B)$ between sets A and B from the family $CL(X)$ of all nonempty closed subsets of X is defined as

$$\rho(A, B) = \sum_{i=1}^{\infty} \frac{1}{2^i} \frac{|d(y_i, A) - d(y_i, B)|}{1 + |d(y_i, A) - d(y_i, B)|},$$

where $d(y_i, A)$ is the distance from a point $y_i \in Y$ to the set A . This distance is independent of the choice of the set Y , and the function $\rho(A, B)$ is a metric on the space $CL(X)$. The convergence of a sequence of sets A_n , $n \geq 1$, from the metric space $(CL(X), \rho(\cdot))$ is equivalent to the Kuratowski convergence of this sequence. We prove the completeness and separability of the space $(CL(X), \rho(\cdot))$ and give necessary and sufficient conditions for the compactness of sets in this space. The space $C(\mathcal{T}, CL(X))$ of all continuous mappings from \mathcal{T} to $(CL(X), \rho(\cdot))$ is endowed with the topology of uniform convergence on compact sets from \mathcal{T} . We prove the completeness and separability of the space $C(\mathcal{T}, CL(X))$ and give necessary and sufficient conditions for the compactness of sets in this space. These results are reformulated for the space $C(\mathcal{T}, CCL(X))$, where $\mathcal{T} = [0, 1]$, X is a finite-dimensional Euclidean space, and $CCL(X)$ is the space of all nonempty closed convex sets from X with the metric $\rho(\cdot)$. This space plays a crucial role in the study of sweeping processes. A counterexample showing the significance of the assumption of the compactness of closed balls from X is given.

Keywords: unbounded sets, Kuratowski convergence, compactness.

REFERENCES

1. Tolstonogov A.A. Investigation of a new class of control systems. *Dokl. Math.*, 2012, vol. 85, no. 2, pp. 178–180. doi: 10.1134/S1064562412020056.
2. Tolstonogov A.A. Control sweeping processes. *J. Convex Analysis*, 2016, vol. 23, no. 4, pp. 1099–1123.
3. Shouchuan Hu, Papageorgiou N.S. *Handbook of multivalued analysis. Theory. Vol. 1*. Ser. Math. Its Appl., vol. 149, Dordrecht, Boston, London: Kluwer, 1997, 964 p. ISBN: 0792346823.
4. Panasenko E.A., Rodina L.I., Tonkov E.L. The space $clcv(R^n)$ with the Hausdorff-Bebutov metric and differential inclusions. *Proc. Steklov Inst. Math. (Suppl.)*, 2011, vol. 275, suppl. 1, pp. 121–136. doi: 10.1134/S0081543811090094.
5. Zhukovskiy E.S., Panasenko E.A. On multivalued maps with images in the space of closed subset of a metric space. *Fixed Point Theory. Appl.*, 2013, no. 10, 21 p. doi: 10.1186/1687-1812-2013-10.
6. Tolstonogov A.A. Compactness in the space of set-valued mappings with closed values. *Dokl. Math.*, 2014, vol. 89, no. 3, pp. 293–295. doi: 10.1134/S1064562414030120.
7. Bourbaki N. *Éléments de Mathématique, Première partie, Livre III, volume Topologie Générale*. Paris: Hermann, 1960, 366 p. ISBN: 2903684002X. Translated to Russian under the title *Obshchaya topologiya. Ispol'zovanie veshchestvennykh chisel v obshchei topologii. Funktsional'nye prostranstva. Svodka rezul'tatov*. Moscow, Nauka Publ., 1975, 408 p.
8. Kuratowski K. *Topology. Vol. I*. N Y, London: Acad. Press, 1966, 560 p. ISBN: 978-0-12-429201-7. Translated to Russian under the title *Topologiya. T. 1*. Moscow, Mir Publ., 1966, 594 p.

9. Beer G. Metric spaces with nice closed balls and distance functions for closed sets. *Bull. Australian Math. Soc.*, 1987, vol. 35, no. 1, pp. 81–96. doi: 10.1017/S000497270001306X.
10. Bourbaki N. *Eléments de mathématique, Fascicule II, Livre III, Topologie générale, Chap. 1, Structures topologiques, Chap. 2, structures uniformes*. Paris: Hermann, 1965, 255 p.
ISBN(1971 ed.): 3-540-33936-1. Translated to Russian under the title *Obshchaya topologiya. Osnovnye struktury*. Moscow, Nauka Publ., 1968, 275 p.
11. Kuratowski K. *Topology. Vol. II*. N Y, London: Acad. Press, 1968, 608 p. ISBN: 978-0-12-429202-4.
Translated to Russian under the title *Topologiya. T. 2*. Moscow, Mir Publ., 1969, 624 p.
12. Beer G. On convergence of closed sets in a metric space and distance functions. *Bull. Australian Math. Soc.*, 1985, vol. 31, pp. 421–432. doi: 10.1017/S0004972700009370.
13. Matheron G. *Random sets and integral geometry*. New York: Wiley, 1975, 261 p.
ISBN: 978-0-471-57621-1. Translated to Russian under the title *Sluchainye mnozhestva i integral'naya geometriya*. Moscow, Mir Publ., 1978, 318 p.

The paper was received by the Editorial Office on September 25, 2017.

Aleksandr Aleksandrovich Tolstonogov, RAS Corresponding Member, Dr. Phys.-Math. Sci., Prof., Matrosov Institute for System Dynamics and Control Theory of Siberian Branch of Russian Academy of Sciences, Irkutsk, 664033 Russia, e-mail: aatol@icc.ru.

Cite this article as:

A. A. Tolstonogov. Space of continuous set-valued mappings with closed unbounded values, *Trudy Inst. Mat. Mekh. UrO RAN*, 2018, vol. 24, no. 1, pp. 200–208.