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ON ASYMPTOTIC PROPERTIES OF SOLUTIONS OF CONTROL SYSTEMS WITH RANDOM PARAMETERS

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Differential equations and control systems with impulse action and random parameters are studied. These objects are characterized by stochastic behavior: the lengths θ_k of the intervals between the times of the impulses τ_k, $k = 0, 1, \ldots$, are random variables and the magnitudes of the impulses also depend on random actions. The basic object of research is the control system

$$\dot{x} = f(t, x, u), \quad t \neq \tau_k,$$
$$\Delta x\big|_{t=\tau_k} = g(x, w_k, v_k),$$

which depends on random parameters $\theta_k = \tau_{k+1} - \tau_k$ and v_k, $k = 0, 1, \ldots$. A probability measure μ is defined on the set Σ of all possible sequences $\big((\theta_0, v_0), \ldots, (\theta_k, v_k), \ldots\big)$. Admissible controls $u = u(t)$ are bounded measurable functions with values in a compact set $U \subset R^m$, and the vector w_k is also a control affecting the behavior of the system at the times τ_k. We consider the set $\mathfrak{M} = \big\{(t, x) : t \in [0, +\infty),\ x \in M(t)\big\}$ defined by the function $t \mapsto M(t)$, which is continuous in the Hausdorff metric. The main result of the paper is sufficient conditions for the Lyapunov stability and asymptotic stability of the set $\mathfrak{M}$ with probability one. It is shown that the stability analysis of a set by means of the method of Lyapunov functions can be reduced to studying the stability of the zero solution of the corresponding differential equation. We also study the asymptotic behavior of solutions of differential equations with impulse action and random parameters. Conditions are obtained under which the solutions possess the Lyapunov stability and asymptotic stability for all values of the random parameter and with probability one. The results are illustrated by a probability model of a population subject to harvesting and by a model of competition of two kinds with impulse action.

Keywords: differential equations and control systems with random parameters, Lyapunov stability, asymptotic stability.

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