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ON SOME PROPERTIES OF VECTOR MEASURES

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We study the properties of a parameterized sequence of countably additive vector measures having a density, defined on a compact space with a nonnegative nonatomic Radon measure, and taking values in a separable Banach space. Each vector measure in this sequence depends continuously on a parameter belonging to some metric space. It is assumed that a countable locally finite open covering and a partition of unity inscribed in it are given in the metric space of the parameters. It is proved that, in the compact support space of the vector measures (with Radon measure), for each value of the parameter, there exists a sequence of measurable (with respect to the Radon measure on the support space of the vector measures) subsets of this compact space that forms a partition of this space. Moreover, the sequence of measurable partitions depends uniformly continuously on the parameter and, for each value of the parameter and for each value of the index of the sequence of measures, the relative value of the measure of the corresponding subset of the partition of the compact space can be approximated uniformly by the corresponding value of the partition function of unity.

Keywords: Lyapunov theorem, countably additive vector measure, density of a vector measure, partition of unity, continuous mapping.

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