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# CONSTRUCTION OF A STRONG NASH EQUILIBRIUM IN A CLASS OF INFINITE NON-ZERO-SUM GAMES

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In our previous papers (2002, 2017), we derived conditions for the existence of a strong Nash equilibrium in multistage non-zero-sum games under additional constraints on the possible deviations of coalitions from their agreed-upon strategies. These constraints allowed only one-time simultaneous deviations of all the players in a coalition. However, it is clear that in real-world problems the deviations of different members of a coalition may occur at different times (at different stages of the game), which makes the punishment strategy approach proposed by the authors earlier inapplicable in the general case. The fundamental difficulty is that in the general case the players who must punish the deviating coalition know neither the members of this coalition nor the times when each player performs the deviation. In this paper we propose a new punishment strategy, which does not require the full information about the deviating coalition but uses only the fact of deviation of at least one player of the coalition. Of course, this punishment strategy can be realized only under some additional constraints on simultaneous components of the game in an infinite-stage game. Under these additional constraints it was proved that the punishment of the deviating coalition can be effectively realized. As a result, the existence of a strong Nash equilibrium was established.

Keywords: strong Nash equilibrium, characteristic function, multistage game, repeated game, imputation, core.

## REFERENCES

1. Krasovskii N.N., Subbotin A.I. *Game-theoretical control problems*. N Y, Springer, 1988, 517 p. ISBN: 978-1-4612-8318-8 . Original Russian text published in Krasovskii N.N., Subbotin A.I. *Pozitsionnye differentsial'nye igry*. Moscow, Fizmatlit Publ., 1974, 456 p.
2. Petrosyan L.A. Signal strategies and behavior strategies in one class of infinite positional games. In: *Positional games, collection of articles*, Vorobyova N.N., Rublevskaya I.N. (eds.), Moscow, Nauka Publ., 1967, pp. 221–230 (in Russian).
3. Subbotin A.I., Chentsov A.G. *Optimizatsiya garantii v zadachakh upravleniya* [Optimization of guarantee in control problems]. Moscow, Nauka Publ., 1981, 288 p.
4. Aumann R.J., Maschler M. *Repeated games with incomplete information*. Cambridge, MIT Press, 1995, 360 p. ISBN: 9780262011471 .
5. Fudenberg D., Maskin E. The Folk theorem in repeated games with discounting or with incomplete information. *Econometrica*, 1986, vol. 54, no. 3, pp. 533–554. doi: 10.2307/1911307 .
6. Maschler M., Solan E., Zamir S. *Game theory*. Cambridge, Cambridge University Press, 2013, 1003 p. ISBN: 978-1-107-00548-8 .
7. Myerson R.B. Multistage games with communication. *Econometrica*, 1986, vol. 54, no. 2, pp. 323–358. doi: 10.2307/1913154 .
8. Nash J. Non-cooperative games. *Ann. Mathematics*, 1951, vol. 54, no. 2, pp. 286–295. doi: 10.2307/1969529 .
9. Neumann J., Morgenstern O. *Theory of games and economic behavior*. Princeton, 1947, 641 p. doi: 10.1177/1468795X06065810 .
10. Petrosjan L.A., Grauer L.V.. Strong Nash equilibrium in multistage games. *International Game Theory Review*, 2002, vol. 4, no. 3, pp. 255–264. doi: 10.1142/S0219198902000689 .

11. Petrosyan L., Chistyakov S., Pankratova Ya. Existence of strong Nash equilibrium in repeated and multistage games. *Constructive Nonsmooth Analysis and Related Topics* (*Dedicated to the Memory of V.F. Demyanov*), *CNSA 2017, Saint-Petersburg*, p. 255–257. doi: 10.1109/CNSA.2017.7974003 .
12. Rubinstein A. Equilibrium in supergames. *Essays in Game Theory*, 1994, pp. 17–28. doi: 10.1007/978-1-4612-2648-2_2 .

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