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## A MULTIPLE CAPTURE IN A GROUP PURSUIT PROBLEM WITH FRACTIONAL DERIVATIVES

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In a finite-dimensional Euclidean space, we consider a problem of pursuing one evader by a group of pursuers with equal capabilities of all participants. The dynamics of the problem is described by the system

$$D^{(\alpha)}z_i = az_i + u_i - v, \quad u_i, v \in V,$$

where  $D^{(\alpha)}f$  is the Caputo derivative of order  $\alpha \in (1, 2)$  of the function  $f$ . The set of admissible controls  $V$  is a strictly convex compact set and  $a$  is a real number. The aim of the group of pursuers is to catch the evader by at least  $m$  different pursuers, possibly at different times. The terminal sets are the origin. The pursuers use quasi-strategies. We obtain sufficient conditions for the solvability of the pursuit problem in terms of the initial positions. The investigation is based on the method of resolving functions, which allows us to obtain sufficient conditions for the termination of the approach problem in some guaranteed time.

Keywords: differential game, group pursuit, multiple capture, pursuer, evader.

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