ON AN OPTIMAL CONTROL PROBLEM WITH DISCONTINUOUS INTEGRAND

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We consider an optimal control problem for an autonomous differential inclusion with free terminal time and a mixed functional which contains the characteristic function of a given open set $M \subset \mathbb{R}^n$ in the integral term. The statement of the problem weakens the statement of the classical optimal control problem with state constraints to the case when the presence of admissible trajectories of the system in the set M is physically allowed but unfavorable due to safety or instability reasons. Using an approximation approach, necessary conditions for the optimality of an admissible trajectory are obtained in the form of Clarke's Hamiltonian inclusion. The result involves a nonstandard stationarity condition for the Hamiltonian. Conditions guaranteeing their nondegeneracy and pointwise nontriviality are presented. The results obtained extend the author's previous results to the case of a problem with free terminal time and more general functional.

Keywords: risk zone, state constraints, optimal control, Hamiltonian inclusion, Pontryagin maximum principle, nondegeneracy conditions.

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